UNIVERSITY OF CAMBRIDGE

# MATHEMATICAL TRIPOS Part III

Monday 7 June, 2004 1.30 to 4.30

## PAPER 18

## SPECTRAL GEOMETRY

*Attempt* **THREE** *questions.*

*There are* **four** *questions in total.*

*The questions carry equal weight.*

*You may use without proof results from other areas of Mathematics, such as Analysis or Topology, provided you state them clearly.*

**You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.**

**1** State and prove what is the spectrum of a flat torus. Prove that two isospectral tori of dimension two are isometric.

**2** Define the heat kernel and heat invariants for the Laplacian acting on functions on a Riemannian manifold $M$. State some geometric invariants that are determined by the heat invariants.

Prove that the heat kernel is defined, and has an appropriate asymptotic expansion, on a neighbourhood of the diagonal in $M \times M$.

**3** State Sumada's Theorem concerning certain coverings and resulting isospectral manifolds. Explain, with proof, how to detect whether these manifolds are isometric.

Give an example of a group $T$ with non-isomorphic Gassman subgroups. Deduce the existence, for any $n \in \mathbb{N}$, of $n$ manifolds of dimension 5 all of which have the same spectrum but no two of which are isometric.

**4** Identify a pair of Gassman subgroups $U_1, U_2$ of $PSL(3,2)$. Explain *briefly* why they are Gassman equivalent and not conjugate.

Assuming any result you require about hyperbolic tri-rectangles, construct an infinite family, parameterised by an interval on the real line, of examples of isospectral non-isometric Riemann surfaces of genus 4.

[You may use the fact that $PSL(3,2)$ has generators $A$ and $D$ with commutator $C = [D, A]$ of order 7 such that the permutation actions of $A$ and $D$ on the cosets $U_iC^n$ are given by

| Action of | On cosets of $U_1$ | On cosets of $U_2$ |
|---|---|---|
| A | (0)(125)(364) | (0)(143)(256) |
| D | (1)(03)(2645) | (4)(25)(0163) |

where, in each case, the coset $U_iC^n$ is denoted by $n$. ]