UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 3 June, 2004 9 to 12

PAPER 13

TOPICS IN COMBINATORICS

Attempt **THREE** *questions.*

There are **five** *questions in total.*

The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 State and prove the discrete Poincaré inequality. Use it to prove that there is a polynomial-time algorithm for approximating, with high probability, the permanent of a suitably dense 01-matrix.

2 State and prove Szemerédi's regularity lemma, and discuss its applications.

3 Proving any preliminary results you might need (except that you may assume an upper bound on the number of edges in a planar graph), show that n points in the plane must determine at least $cn^{4/5}$ distinct distances, for some absolute constant c.

4 Assuming a result relating monotone circuit complexity to approximation of lattices, obtain a superpolynomial lower bound for the monotone circuit complexity of the clique function.

5 Write an essay on the Frankl-Wilson theorem and its applications.