UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 3 June 2003 9 to 11

PAPER 68

COMPUTER AIDED GEOMETRIC DESIGN

Attempt **FOUR** *questions.*

There are **six** *questions in total.*

The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 AB and CD are line segments in Euclidean space of 3 dimensions. Define an algorithm for determining points P (on AB) and Q (on CD) such that the distance PQ is a minimum. You may assume that the endpoints A, B, C and D of the two segments are in general position.

What particular configurations would need to be considered, if the endpoints could not be assumed to be in general position ?

2 What are the Bernstein polynomials of degree n?

A Bezier curve is a parametric curve whose equation is

$$P(t) = \sum_{i=0}^{n} P_i f_i(t)$$

where the P_i are the control points and the f_i are Bernstein polynomials.

Show that the piece of this curve lying between $t = 0$ and $t = 1$ lies inside the convex hull of the control points, and that the first derivative of the curve can be expressed as a similar combination of the first differences of the control points and the Bernstein polynomials of one degree lower.

3 What are the enquiries which can be made of a general parametric curve ?

Define one algorithm to compute, in terms of those enquiries, the intersection of a general subdivision curve with a given plane. What are the strengths and weaknesses of your chosen algorithm ?

4 What are the enquiries which can be made of a general subdivision curve ?

Define an algorithm to compute, in terms of those enquiries, the intersections of a general parametric curve with a given plane.

How is good speed obtained in such an algorithm ?

5 How are the univariate box-splines defined ?

Show that the box-spline basis functions are

– of finite support

– positive within that support

– sum to unity

How do the

– level of continuity

– support width

– degree of the pieces

vary with the dimension of the box projected ?

In the bivariate case, how do box-splines get defined over a variety of different grids ?

6 A particular refinement scheme takes a manifold configuration of vertices, edges and facets, and computes a new one by creating a new vertex at the midpoint of every old edge, a new facet within each old facet by joining, in sequence, the new vertices on the old facet's edges, and a new facet across each old vertex by joining, in sequence, the new vertices on the edges radiating from the old vertex.

Assuming that the original vertices are in general position, show that the limit surface of this refinement

– is manifold

– lies within the convex hull of the original vertices

– contains the centroid of the vertices of each of the original facets.

– consists of parametric quadratic triangle pieces

– is C1 everywhere except at a number of points not exceeding the sum of the numbers of original vertices and facets

– is C1 everywhere

Accurate identification of how these properties may be proved will be sufficient, rather than detailed proofs.