UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 10 June 2003 9 to 11

PAPER 53

SUPERSYMMETRY

Answer **ALL** *questions in Section A.*

Answer any **ONE** *of the three questions in Section B.*

Sections A and B carry equal credit.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

Section A

1 You are given that

$$\gamma_{pq}\gamma^m\gamma^{pq} = \alpha\gamma^m$$

for Dirac matrices in spacetime dimension D. What is the constant α as a function of D?

2 You are given that λ is an anticommuting Majorana spinor of $Spin(3,1)$, with charge conjugate $\bar{\lambda}$. Which of the following tensors vanishes identically?

$$(a)\ \bar{\lambda}\gamma^m\lambda, \qquad (b)\ \bar{\lambda}\lambda, \qquad (c)\ \bar{\lambda}\gamma^{mn}\gamma_5\lambda, \qquad (d)\ \bar{\lambda}\gamma^{mnp}\lambda.$$

3 Given that $\{Q^\alpha, Q^\beta\} = 2(\gamma_m C)^{\alpha\beta}P^m$, for Majorana spinor charge Q and 4-momentum P^m, prove that any state annihilated by one linear combination of the components of Q is annihilated by either two or four linearly independent combinations.

4 Write down the spin content content of the superspin $\frac{1}{2}$ unitary irrep of the super-Poincaré group. How does it decompose into massless superhelicity irreps?

5 You are given that $V(x,\theta)$ is a real general superfield. Why is $\frac{\partial V}{\partial\theta}$ not a superfield? How is this difficulty circumvented?

6 Let Z be a chiral superfield. Write down the most general superspace action for Z for which the component field equations are partial differential equations of order $\leqslant 2$. State without proof how this action must be restricted in order to ensure renormalizability (in four spacetime dimensions).

7 The differential 1-forms $(dX^m, d\theta^\alpha)$ (for Majorana spinor θ) consitute a basis of 1-forms on superspace. Write down an alternative, supertranslation-invariant, basis. Hence deduce that the 3-form

$$H = \left(dX^m + i\bar{\theta}\gamma^m d\theta\right) d\bar{\theta}\gamma_m d\theta$$

is super-Poincaré invariant. Stating without proof any gamma-matrix identity that you may need, show that $dH = 0$.

Section B

8 Write an essay with the title *Spontaneous breaking of supersymmetry.* Your essay should include a discussion of the Witten index and the implications of Goldstone's theorem, and should be illustrated with examples from supersymmetric quantum mechanics and field theories of chiral superfields.

9 Write an essay with the title *Gauge theories in superspace.* Your essay should include a discussion of the superspace analogs of the Maxwell gauge potential, gauge transformations, field strength and Bianchi identity. You should also discuss the minimal coupling to chiral superfields, and include a brief discussion of the non-abelian generalization.

10 Write an essay with the title *Supermembranes.* Your essay should include a discussion of domain walls in the Wess-Zumino model, Dirac's membrane action as an effective worldvolume field theory, and a discussion of its supersymmetric extension, including the need for 'kappa-symmetry'.