UNIVERSITY OF CAMBRIDGE

# MATHEMATICAL TRIPOS Part III

Thursday 29 May 2003 9 to 12

## PAPER 19

## SET THEORY

*Attempt* **FOUR** *questions.*

*There are* **eight** *questions in total.*

*The questions carry equal weight.*

**You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.**

**1** What is a WQO? State and prove the perfect subsequence lemma. **Either**

(i) Prove Kruskal's theorem and Friedman's Finite Form, **or**

(ii) An **incline** is a structure with two associative and commutative binary operations $+$ and $\cdot$ satisfying

$(\forall xyz)(x \cdot (y+z) = x \cdot y + x \cdot z)$;

$(\forall x)(x + x = x)$;

$(\forall xy)(x + xy = x)$.

We define a relation $\leqslant$ by $x + y \leqslant x$.

Prove that $\leqslant$ is a quasi-order.

Let $(I, +, \cdot)$ be a finitely generated incline. Show that $(I, \leqslant)$ is a WQO.

**2** Write an essay on large cardinals.

**3** Prove the independence of the axiom of foundation, and extend your technique to prove the independence of the axiom of choice from ZF minus foundation.

**4** What are inner models, and what can they be used to prove?

**5** Exhibit a recursive partition of $[\mathbb{N}]^n$ with no recursive monochromatic set. Prove the Erdös-Rado theorem on the existence of uncountable monochromatic sets for partitions of $n$-tuples. What can you say about infinite exponent partition relations?

**6** What is a saturated model? Using ultraproducts or otherwise, state and prove a theorem about the existence of saturated models.

By exploiting saturated models, or otherwise, prove the consistency of NFU.

**7** (i) Prove the Ehrenfeucht-Mostowski theorem.

(ii) What is AD, the axiom of determinacy? Which games can you prove to be determined? Establish that AD is inconsistent with AC.

**8** Prove Fodor's theorem and the Cantor Normal Form theorem.