UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 29 May 2003 9 to 12

PAPER 15

BASIC ALGEBRAIC GEOMETRY

Attempt **THREE** *questions.*

There are **five** *questions in total.*

The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Prove carefully that a nonsingular cubic surface contains a line. You may state without proof any results from the dimension theory of morphisms of algebraic varieties.

2 Let $X \subset \mathbf{P}^3$ be the rational normal curve in $\mathbf{P}^3$; in other words, X is the image of the morphism

$$\mathbf{P}^1 \ni (s,t) \to (s^3, s^2t, st^2, t^3) \in \mathbf{P}^3$$

Prove that the homogeneous ideal of X is the ideal

$$I = (x_0x_3 - x_1x_2, x_0x_2 - x_1^2, x_1x_3 - x_2^2)$$

generated by the three obvious quadrics containing X.

3 Let A be a unique factorization domain, K its field of fractions. Let $K \subset L$ be an algebraic extension of fields. Then if $b \in L$ is integral over A, the norm $N_K^L(b)$ is an element of A. Discuss briefly the relevance of this statement to the dimension theory of algebraic varieties.

4 Write an essay on coherent cohomology. Your essay should at least contain the definition of Čech cohomology, state some basic theorems on the cohomology of coherent sheaves on projective space and projective varieties, and discuss some examples.

5 State briefly the axiomatic properties of Chern classes of vector bundles on (nonsingular) algebraic varieties.

If Y is a nonsingular algebraic variety, denote as usual by Ω^1_Y the cotangent bundle of Y (this is the same as the bundle of Kaehler differentials). Let $X_d \subset \mathbf{P}^n$ be a nonsingular hypersurface of degree d in $\mathbf{P}^n$; calculate the Chern classes $c_i(\Omega^1_X)$ in terms of $h = c_1(\mathcal{O}_X(1))$.

[*Use the exact sequences* $0 \to \Omega^1_{\mathbf{P}} \to \mathcal{O}_{\mathbf{P}}(-1)^{n+1} \to \mathcal{O}_{\mathbf{P}} \to 0$, *and* $0 \to \mathcal{O}_X(-d) \to \Omega^1_{\mathbf{P}|X} \to \Omega^1_X \to 0$. *Apply the Whitney formula.*]