UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 29 May 2003 1.30 to 4.30

PAPER 11

TOPOLOGICAL GROUPS

Attempt **THREE** *questions.*

There are **four** *questions in total.*

The questions carry equal weight.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Show that a topological group is metrisable with a left invariant metric if and only if it has a countable base of neighbourhoods at the identity. (Any properties of neighbourhoods that you need should be proved.)

Give, with proof, an example of a metrisable topological group which does not have a left and right invariant metric.

2 Show that any compactly generated, metrisable group has a Haar measure.

[You may assume the existence of functions with the properties needed by your proof.]

3 Explain and prove the identification between group characters and the characters of Gelfand theory for $L^1(G)$ where G is a locally compact Abelian Hausdorff group.

4 State and prove Bochner's theorem on positive definite functions. You may assume any elementary results on positive definite functions that you need.