UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 3 June 2002 1.30 to 3.30

PAPER 70

YANG-MILLS FIELD THEORY

*Attempt **THREE** questions*

*There are **four** questions in total*

The questions carry equal weight

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Find the field equations and the conserved energy and momentum for the $1+1$ dimensional scalar field theory with Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi \ - \ \frac{1}{2}\lambda^2(\phi^2-1)^2.$$

Find the static kink solution of this theory. What is the mass of the kink?

By using a Lorentz boost, or otherwise, find the solution describing a kink moving with velocity v $(|v|<1)$. Calculate the momentum of the moving kink and verify that it is consistent with the interpretation of the kink as a relativistic particle.

2 Write an essay on the Bogomolny equations for **either** (i) Yang-Mills-Higgs monopoles, **or** (ii) Vortices in the Abelian Higgs model.

You should include some discussion of the solutions of the equations.

3 Discuss the notion of topological degree, and how it is used to classify solitons.

4 Write an essay on Chern numbers and Chern-Simons numbers, in the context of pure $SU(2)$ Yang-Mills theory defined in three or four dimensions.