UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday 7 June 2002 9 to 11

PAPER 68

SUPERSYMMETRY

Attempt question ***ONE***

and any ***TWO*** *of questions 2, 3, and 4*

There are ***four*** *questions in total*

The questions carry equal weight

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Provide a short answer to each of the following questions in *no more* than 15 lines each:

(a) Give three independent reasons that favour the possible existence of supersymmetry at energies close to $1TeV$. If supersymmetry is not discovered at these energies, give two reasons to argue for why it is still relevant.

(b) How do we label one-particle-state representations of the Poincaré group? How does that change with $N = 1$ supersymmetry? Provide the labels (only the labels) of the different states inside an arbitrary multiplet for both massive and massless states.

(c) Why is extended supersymmetry not expected to be relevant at low energies?

(d) Explain the difference between the supersymmetric realization of the standard Higgs effect and the super-Higgs effect.

(e) Why do we say that supersymmetry is a spacetime symmetry?

(f) Is any function of anticommuting variables a superfield? Explain with examples.

(h) Name the functions and constants that must be specified in order to completely determine the $N = 1$ (global or local) supersymmetric action for arbitrary chiral and vector superfields. Mention briefly which of the functions are holomorphic and how they behave under quantum corrections.

2 Use the extended supersymmetry algebra to derive the BPS bound on massive states. Which properties characterise BPS states?

(*Hint: You may find useful to consider the anticommutator*
$\left\{Q_\alpha^A - \Gamma_\alpha^A, \bar{Q}_{\dot{\beta}A} - \bar{\Gamma}_{\dot{\beta}A}\right\}$ *with* $\Gamma_\alpha^A \equiv \epsilon_{\alpha\beta} U^{AB} \bar{Q}_{\dot{\gamma}B} \left(\bar{\sigma}^0\right)^{\dot{\gamma}\beta}$ *and* U *unitary.*)

Use the $N = 1$ supersymmetry algebra to prove that:

(a) In every supermultiplet, the number of bosons equals the number of fermions

(b) The energy is non-negative and supersymmetry is broken if the energy of the vacuum is strictly positive.

(*Hint: consider* $\mathrm{Tr}\left[(-1)^F \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\}\right]$).

3 Consider $N = 1$ supergravity with three chiral superfields S, T, C. The Kähler potential is in Planck units:

$$K = -\log(S + S^*) - 3 \log(T + T^* - C^* C)$$

The superpotential is:

$$W = C^3 + a\, e^{-\alpha S} + b$$

where a, b are arbitrary complex numbers and $\alpha > 0$. Compute the scalar potential. Find the auxiliary field for S, T, C and verify that supersymmetry is broken. What is the value of the vacuum energy at its minimum? Are there flat directions? What is the gravitino mass?

(*Hint: In supergravity the auxilary fields are proportional to the Kähler covariant derivatives* $DW = \partial W/\partial\Phi + W \partial K/\partial\Phi$.)

4 Consider the field content of the MSSM, transforming under $SU(3) \times SU(2) \times U(1)_Y$ as:

$$\begin{array}{lll} Q_i = (3, 2, -\frac{1}{6}), & U_i^R = (\bar{3}, 1, \frac{2}{3}), & D_i^R = (\bar{3}, 1, -\frac{1}{3}), \\ L_i = (1, 2, \frac{1}{2}), & E_i^R = (1, 1, -1), & N_i^R = (1, 1, 0), \\ H_1 = (1, 2, \frac{1}{2}), & H_2 = (1, 2, -\frac{1}{2}), & \end{array}$$

where the indices $i = 1, 2, 3$ label the three different generations and the last Higgs superfield is introduced to avoid anomalies.

Write down the most general cubic superpotential for these fields, invariant under the symmetries of the standard model. Separate the terms that preserve baryon and lepton number from those that do not preserve it. Show that combining two of the baryon/lepton number violating terms will induce proton decay: $p \to e^+ + \pi^0$. Estimate the decay rate of the proton via this channel based on dimensional grounds.

The experimental lower bound on the proton lifetime is approximately:

$$\tau_{\text{proton}} > 5.5 \times 10^{32} \text{yrs} = 1.6 \times 10^{40} \text{sec} = 2.4 \times 10^{64} \text{GeV}^{-1}$$

Use this to determine an upper bound on the product of the two 'Yukawa' couplings that give rise to proton decay above.

Verify that R-parity forbids all baryon/lepton number violating terms while preserving the fermion mass terms.