UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS   Part III

Thursday 6 June 2002   1.30 to 3.30

PAPER 59

COMPUTER AIDED GEOMETRIC DESIGN

Candidates should answer **FOUR** *questions.*

There are **six** *questions in total*

The questions carry equal weight

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1   Define the width, and arity of a univariate subdivision scheme in terms of the subdivision matrix, and define the support and functional precision set of the scheme.

Determine the support, functional precision set and level of derivative continuity of the univariate binary scheme whose mask is $[1, 2, 1]/2$.

2   Identify the methods which a parametric curve object should support in order to allow efficient interrogation.

What is the equation satisfied by an interior point, $P(t)$, of a given parametric curve, which is a locally closest point to a given point Q ?

Write an efficient algorithm for finding a nearest point on a given subdivision curve to a given point.
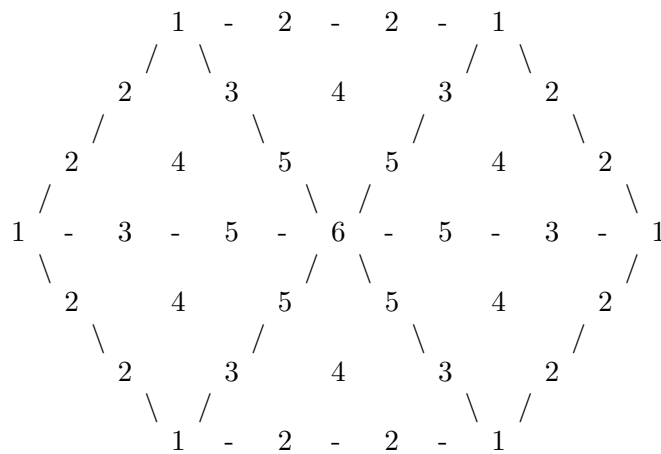
3   What is a Solid Body transform ? How can it be represented for computational purposes, and what constraints are satisfied by its coefficients ?

Prove that if the control points of a Bezier cubic curve are transformed by a Solid Body transform, every point on the curve is transformed by the same transformation.

For how wide a class of transforms can similar results be proved, and for how wide a class of parametric objects?

4   What are lateral artifacts of a bivariate subdivision scheme ?

Does the ternary scheme with mask

```
            1    -   2    -   2    -   1
           / \             / \
          2       3       4       3       2
         /             \         /             \
        2       4       5       5       4       2
       /             \     /             \
      1   -   3   -   5   -   6   -   5   -   3   -   1
       \             /     \             /
        2       4       5       5       4       2
         \             /         \             /
          2       3       4       3       2
           \   /                 \   /
            1    -   2    -   2    -   1
```

(all divided by 12) have lateral artifacts ?

If necessary, modify it so that lateral artifacts are not present. What then is the associated univariate scheme appropriate for boundary curves ?

5   What is a tensor product surface definition ?

Prove that a tensor product surface based on piecewise polynomials inherits:-
              –   positivity
              –   level of derivative continuity
       and –   summation to unity

from its factors.

Give an example of a parametric surface definition which is not a tensor product.

6   Identify the methods which a subdivision surface object should support in order to allow efficient interrogation.

Write an algorithm for finding the curve in which the subdivision surface is cut by a given plane. What are the issues to be considered in implementation of the methods making the code efficient?