UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Thursday 30 May 2002 9 to 12

PAPER 21

SET THEORY

*Full marks may be obtained by complete answers to **FOUR** questions*

*There are **six** questions in total*

The questions carry equal weight

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Prove Kruskal's theorem and Friedman's Finite Form.

2 Prove the independence of the axiom of foundation, and extend your technique to prove the independence of the axiom of choice from ZF minus foundation.

3 What are inner models, and what can they be used to prove?

4 Exhibit a recursive partition of $[\mathbb{N}]^3$ with no recursive monochromatic set. Prove the Erdös-Rado theorem on the existence of uncountable monochromatic sets for partitions of n-tuples. What can you say about infinite exponent partition relations?

5 What is a saturated model? Using ultraproducts or otherwise, state and prove a theorem about the existence of saturated models. Prove the consistency of NFU.

6 (a) Prove the Ehrenfeucht-Mostowski theorem.

(b) What is AD, the axiom of determinacy? Which games can you prove to be determinate? Establish that AD is inconsistent with AC.