

MATHEMATICAL TRIPOS      Part III

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Tuesday 4 June 2002    1.30 to 4.30

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PAPER 13

CURVES AND THEIR JACOBIANS

*Attempt **THREE** questions*

*There are **five** questions in total*

*The questions carry equal weight*

You may not start to read the questions  
printed on the subsequent pages until  
instructed to do so by the Invigilator.

Throughout,  $X$  will denote a smooth projective complex curve and  $X^{(d)}$  its  $d$ 'th symmetric product.

**1** State and prove the Riemann bilinear relations. Explain their significance in the construction of the Jacobian  $Jac(X)$  of  $X$ .

**2** State and prove Abel's theorem. (Any result about the existence and structure of the quotient of an algebraic variety by a finite group may be assumed.)

**3** State and prove a theorem that describes the derivative of the Abelian sum map  $u_d : X^{(d)} \rightarrow Jac(X)$ .

**4** Define Riemann's theta function and prove a theorem that relates its zero locus to the image  $W_{g-1}$  of the map  $u_{g-1}$ . (Questions of convergence need only be treated briefly.)

**5** State and prove Riemann's singularity theorem.