UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Wednesday 13 June 2001 9 to 11

PAPER 66

ADVANCED STRING THEORY

*Attempt **TWO** questions. The questions are of equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Give *brief* answers to the following:

(a) Explain why the endpoints of an open bosonic string can either have Neumann or Dirichlet boundary conditions.

(b) Both endpoints of an open bosonic string satisfy Neumann boundary conditions in $p+1$ directions and Dirichlet in the remainder. Give a physical interpretation of this situation in terms of D-branes.

(c) What is meant by the statement that the D-branes of the superstring are BPS states? How would you demonstrate that the static force between two parallel Dp-branes vanishes?

(d) A collection of D-branes is related to the Yang–Mills–Higgs system. Describe in outline the motivations for this statement. Illustrate this for the system consisting of two parallel D3-branes.

(e) How are D-branes related to solitonic solutions of supergravity?

2 A theory in d space-time dimensions contains a massless $(p+1)$-form potential. Explain how this can couple to extended objects.

List the massless bosonic and fermionic states of the type IIA and type IIB superstring theories in ten dimensions. How does this motivate the existence of a variety of extended objects in these theories and the various charges that they carry?

Give an account of how T-duality relates the type IIA and type IIB theories when one of the spatial directions is a circle of radius R.

3 The bosonic fields of eleven dimensional supergravity have an action that can be written as

$$S = \frac{1}{2\kappa_{11}^2} \int d^{11}x \left(\sqrt{-\det G} \left(R - \frac{1}{2} F^{(4)\,2} \right) - \frac{1}{6} C^{(3)} \wedge F^{(4)} \wedge F^{(4)} \right).$$

Show how the various bosonic terms in the action of ten-dimensional type IIA supergravity arise when the eleven-dimensional theory is compactified on a circle.

How are the various D-branes of type IIA superstring theory interpreted in terms of the eleven-dimensional theory?