UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 12 June 2001 9 to 11

PAPER 65

SUPERSYMMETRY

Attempt ***Question 1*** *and* ***TWO*** *other questions.*

Question 1 *counts for* ***50%*** *of the total marks, the remaining questions are of equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 The answers to each of the following questions should be no more than 15 lines in length:

(i) Give three independent reasons that favour the possible existence of supersymmetry at energies close to 1 TeV. If supersymmetry is not discovered at these energies, give two reasons why it may still be relevant.

(ii) What are BPS states?

(iii) Explain what is meant by the super-Higgs effect.

(iv) Is any function of anticommuting variables θ a superfield? Explain and give examples.

(v) Name the functions and constants that must be specified in order to completely determine the $N = 1$ (global or local) supersymmetric action, up to two derivatives, for arbitrary chiral and vector superfields. Mention briefly which of the functions are holomorphic and how they behave when quantum corrections are taken into account.

2 Write explicitly all the components of the $N = 1$ supersymmetry multiplets corresponding to:

(i) massive particles of maximum spin 1;

(ii) massless particles with maximum helicity 1/2 and 1.

Verify in each case that the number of bosons equals the number of fermions. Is the Higgs mechanism compatible with supersymmetry?

3 Consider a chiral superfield Φ of charge q coupled to an Abelian vector superfield V in a renormalizable Lagrangian. Show that a nonvanishing vacuum expectation value of D, the auxiliary field of V, can break supersymmetry if there is a Fayet-Iliopoulos term. Identify the corresponding goldstino field.

Assuming a vanishing superpotential, write down the D-term part of the scalar potential and find the condition that the Fayet-Iliopoulos term and the charge q have to satisfy for supersymmetry to be broken.

Find the spectrum of this model after supersymmetry is broken. What is the mass splitting of the multiplets?

4 The fields of the MSSM transform under $SU(3) \times SU(2) \times U(1)_Y$ according to

$$\begin{aligned} Q_i &= (3, 2, -\tfrac{1}{6}), & U_i^R &= (\bar{3}, 1, \tfrac{2}{3}), & D_i^R &= (\bar{3}, 1, -\tfrac{1}{3}), \\ L_i &= (1, 2, \tfrac{1}{2}), & E_i^R &= (1, 1, -1), & N_i^R &= (1, 1, 0), \\ H_1 &= (1, 2, \tfrac{1}{2}), & H_2 &= (1, 2, -\tfrac{1}{2}), \end{aligned}$$

where in the notation (n_1, n_2, Y), n_1, n_2 are the dimensions of the representations under $SU(3)$, $SU(2)$ respectively and Y is the hypercharge. The indices $i = 1, 2, 3$ label the three different generations and the last Higgs superfield is introduced to avoid anomalies.

Write down the most general cubic superpotential for these fields, invariant under the symmetries of the standard model. Separate the terms that preserve baryon and lepton number from those that do not preserve it. Show that combining two of the baryon/lepton number violating terms will induce proton decay: $p \to e^+ + \pi^0$. Estimate the decay rate of the proton via this channel based on dimensional grounds.

The experimental lower bound on the proton lifetime is approximately

$$\tau_{\text{proton}} > 5.5 \times 10^{32}\text{yrs.} = 1.6 \times 10^{40}\text{sec.} = 2.4 \times 10^{64}\text{GeV}^{-1}.$$

Use this to determine an upper bound on the product of the two 'Yukawa' couplings that give rise to proton decay.

Verify that R-parity forbids all baryon/lepton number violating terms while preserving the fermion mass terms.