UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 4 June 2001 9 to 12

PAPER 64

STRING THEORY

Attempt ***FOUR*** *questions. The questions are of equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Derive the classical equations of motion for a relativistic string from the Polyakov action, commenting on the invariances of the action and their significance.

2 In the covariant quantization of the Nambu-Goto action, show that there is a physical state of the form $(L_{-2}+\mu L_{-1}^2)|k\rangle$ if the dimension of space-time, $d=26$. What is its norm? Give an outline account of how the No-Ghost Theorem leads to the elimination of negative norm squared states for $d \leqslant 26$.

3 Give a sketch of the path-integral approach to the quantization of the Polyakov action for the bosonic string.

4 Discuss quantization of a closed string when a number of the dimensions of space are compactified to form circles of radius R_j, $1 \leqslant j \leqslant M$. Show that, if any of the radii R_j takes a certain value, the theory acquires additional massless vector particles.

5 Explain how the incorporation of anticommuting fields into string theory leads to the presence in the theory of space-time fermions.