UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Monday 11 June 2001 9 to 11

PAPER 61

QUANTUM INFORMATION PHYSICS

Attempt ***THREE*** *questions. The questions are of equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 The Elitzur-Vaidman quantum scheme for bomb testing can be represented abstractly as follows. A photon qubit is prepared in the state $|0>_p$, and a unitary rotation U is then applied to it. The rotated state then undergoes a brief interaction with a bomb, which may be live or dud, by the following rules:

$$\begin{aligned}
|0\rangle_p \ + \ \text{dud} \quad &\rightarrow \quad |0\rangle_p \ + \ \text{dud}\,, \\
|1\rangle_p \ + \ \text{dud} \quad &\rightarrow \quad |1\rangle_p \ + \ \text{dud}\,, \\
|0\rangle_p \ + \ \text{live} \quad &\rightarrow \quad |0\rangle_p \ + \ \text{live}\,, \\
|1\rangle_p \ + \ \text{live} \quad &\rightarrow \quad \text{explosion}\,.
\end{aligned}$$

If there is no explosion, a further unitary rotation U' is applied to the photon qubit after this interaction. It is then measured by a projective measurement in the basis $|0\rangle_p, |1\rangle_p$. If the bomb did not explode, and if the measurement does not determine with certainty whether the bomb is dud or live, the test is repeated.

Show that, by choosing U, U' appropriately and repeating the test appropriately, the probability of identifying a live bomb without detonating it can be increased to arbitrarily close to $1/2$ within this scheme.

2

Show that any entangled bipartite pure state of two qubits can be transformed, with non-zero probability, to a Bell singlet state

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B\big)\ ,$$

by local measurements and unitary operations at A and/or B.

State the CHSH inequality. (You need not prove it.)

Hence show that the correlations of outcomes of a general sequence of measurements on any entangled bipartite pure state cannot be reproduced by a local hidden variable theory.

3 A and B are separated and share an entangled Werner state

$$W_F = F\,|\Psi_-\rangle\langle\Psi_-| + \tfrac{1}{3}(1-F)\,\big(|\Psi_+\rangle\langle\Psi_+| + |\Phi_+\rangle\langle\Phi_+| + |\Phi_-\rangle\langle\Phi_-|\big)\,,$$

where $F > \frac{1}{2}$ and the Bell states are defined by

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle|1\rangle \pm |1\rangle|0\rangle\big)\,, \qquad |\Phi_\pm\rangle = \frac{1}{\sqrt{2}}\big(|0\rangle|0\rangle \pm |1\rangle|1\rangle\big)\,.$$

Show that it is impossible for them to create a more entangled Werner state $W_{F'}$, with $F' > F$, by local operations and classical communication alone.

4 (i) Let $|\phi_1\rangle, |\phi_2\rangle, \ldots, |\phi_n\rangle$ be states in an n-dimensional Hilbert space with the property that not all of the $|\phi_i\rangle$ are orthogonal. Show that, if quantum theory is universally valid, it is impossible to build a device which, when given an unknown input state which is guaranteed to be one of the $|\phi_i\rangle$, will always correctly identify the state.

(ii) Let $|0\rangle, |1\rangle$ be orthogonal states in a 2-dimensional Hilbert space and $U \in U(2)$ be a unitary operator, with the property that the four states $|0\rangle, |1\rangle, U|0\rangle$ and $U|1\rangle$ are all distinct. Show that, if special relativity and the standard quantum measurement postulate for projective measurements are valid, it is impossible to build a device which, when given an unknown input state which is guaranteed to be one of the four above, will always correctly identify the state.

(iii) Show that, if special relativity and the standard quantum measurement postulate for projective measurements are valid, it is impossible to build a device which, when given two unknown pure states $|\psi_1\rangle$ and $|\psi_2\rangle$ as inputs, is guaranteed to output the value of $\langle\psi_1|\psi_2\rangle$.