UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 12 June 2001 9 to 12

PAPER 39

ATOMIC ASTROPHYSICS

Attempt all ***THREE*** *questions. They are of equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 If a star of radius R_s radiates like a black body of temperature T_s , show that the total energy radiated (per unit time, per unit frequency interval) is $4\pi^2 B_\nu R_s^2$.

Assuming that a planetary nebula contains sufficient hydrogen to absorb all ionizing photons, show (stating any further assumptions you make) that

$$\int N_e \, N_+ \, \beta \, dV = \left(\frac{8 \, \pi^2 \, I_H^3}{h^3 \, c^2} \right) \frac{R_s^2}{y^3} \int_y^\infty \frac{x^2 \, dx}{e^x - 1} \, ,$$

where: N_e , N_+ are the electron, proton number densities; $\beta = \sum_{n=2}^{\infty} \alpha_n$; V represents volume; $y = I_H / kT_s$; I_H is the ionization energy of hydrogen; α_n is the coefficient for recombination to level n.

Use this result to discuss the radii of the ionised regions in a planetary nebula consisting of a uniform mixture of hydrogen and helium, which is of sufficient extent to absorb all ionising photons.

In the case of a non-uniform nebula, indicate briefly how the theory may be used, in conjunction with observed intensities of recombination lines of hydrogen and helium, to deduce the central star temperature T_s.

Discuss, again briefly, the significance of the positions of the central stars of planetary nebulae on the Hertzsprung-Russell diagram, in relation to the theory of stellar evolution.

[*You may assume the Planck intensity law* $B_\nu = 2h \, \nu^3 c^{-2} \, (e^{h\nu/kT} - 1)^{-1}$.]

2 State the Boltzmann equation for the relative populations of the excited states in a plasma in thermodynamic equilibrium, and use it to derive the Saha equation

$$\frac{N_n}{N_e\, N_+} = \frac{\omega_n}{2\,\omega_+} \left(\frac{h^2}{2\pi mkT} \right)^{3/2} \exp(I_n/kT)\,.$$

Hence derive the coefficient $\alpha_d(j, n\ell)$ for dielectronic recombination of an ion X^{+z+1} initially in its ground state i (via the doubly excited state $j, n\ell$ of the ion X^{+z}), in the form

$$\alpha_d(j, n\ell) = 4\,\pi^{3/2}\,\alpha^4 a_0^2\, c\, (E_{ij}/I_H)^{1/2}\, (E_{ij}/kT)^{3/2} \exp(-\bar{E}/kT)\, f_{ij}\, \beta_{j,n\ell}\,,$$

where $\beta_{j,n\ell}$ is a quantity which you should define in terms of autoionisation and radiative transition probabilites.

Discuss briefly the behaviour of the total dielectronic recombination coefficient.

[*You may assume that the Maxwell velocity distribution is*

$$f(v)\,\mathrm{d}v = 4\pi\,(m/2\pi kT)^{3/2}\, v^2\, \exp(-mv^2/\,2kT)\,\mathrm{d}v\,,$$

that the Einstein radiative transition probability is given by

$$A_{ji} = \frac{\alpha^4 c}{2\,a_0} \left(\frac{E_{ij}}{I_H} \right)^2 \frac{\omega_i}{\omega_j}\, f_{ij}$$

and that

$$h^2/\,2\pi m = 4\,\pi\, a_0^2\, I_H\,.]$$

3 UV spectral lines from the ion Si X have been recorded by the SOHO satellite. The transitions $2s^2\, 2p\ {}^2P_{1/2} - 2s\, 2p^2\ {}^2D_{3/2}$ and $2s^2\, 2p\ {}^2P_{3/2} - 2s\, 2p^2\ {}^2D_{5/2}$ give rise to spectral lines at 347 Å and 356 Å respectively. In coronal equilibrium, Si X exists at a temperature of $1.3 \times 10^6 K$.

State the ionisation and recombination processes which are important in the solar corona, and describe the various collisional and radiative excitation and de-excitation processes in a coronal ion such as Si X.

Explain how the theoretical intensities of the spectral lines at 347 Å and 356 Å are calculated and show how the ratio of the observed intensities can be used to derive the average electron density in the emitting plasma.