UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Tuesday 5 June 2001 9 to 11

PAPER 38

MAGNETIC FIELDS IN STARS

Candidates may bring their own notebooks into the examination.

*Candidates should attempt **TWO** questions. The questions are of equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 A conducting fluid rotates about the z-axis of cylindrical polar co-ordinates (s, ϕ, z) with angular velocity $\Omega(s)$. The magnetic field is everywhere perpendicular to the z-axis and can be described by a vector potential $\mathbf{A} = A(s, \phi)\hat{\mathbf{z}}$, where

$$A(s, \phi) = a(s, t)e^{i\psi} \quad \psi = \phi - \Omega(s)t.$$

Show that

$$\frac{1}{\eta}\frac{\partial a}{\partial t} = \Big(\frac{\partial^2 a}{\partial s^2} + \frac{1}{s}\frac{\partial a}{\partial s} - \frac{a}{s^2}\Big) - \Big(\Omega'^2 t^2 a + 2i\Omega' t\frac{\partial a}{\partial s} + i\Omega' t\frac{a}{s} + i\Omega'' ta\Big),$$

where η is the magnetic diffusivity. Hence confirm that $\partial a/\partial t = 0$ if $\eta = 0$. What is the physical significance of this statement.

Now consider the case when

$$\Omega = \begin{cases} \Omega_0(1 - s/d) & s \leqslant d \\ 0 & s > d \end{cases},$$

and there is a uniform magnetic field for $s \gg d$. If the magnetic Reynolds number $R_m = \Omega_0 d^2/\eta \gg 1$ show that for large t the magnetic field in the region $s < d$ decays exponentially as $e^{-(t/\tau_c)^3}$, where $\Omega_0\tau_c \approx (3R_m)^{1/3}$. What is the significance of this result and how does it relate to photospheric magnetoconvection?

2 A rotating star has a magnetic field which drives a stellar wind. The magnetic field $\mathbf{B}$ and the velocity $\mathbf{u}$ outside the star are axisymmetric; they can be decomposed into poloidal and toroidal components so that

$$\mathbf{B} = \Big(-\frac{1}{s}\frac{\partial\chi}{\partial z}, B_\phi(s, z), \frac{1}{s}\frac{\partial\chi}{\partial s}\Big), \quad \mathbf{u} = \mathbf{u}_p + s\Omega(s, z)\hat{\boldsymbol{\phi}}$$

referred to cylindrical polar co-ordinates (s, ϕ, z). Here $\chi(s, z)$ is a Stokes flux function for the poloidal field. Show that the velocity can be expressed in the form

$$\mathbf{u} = \rho^{-1}\kappa(\chi)\mathbf{B} + s\omega(\chi)\hat{\boldsymbol{\phi}},$$

where ρ is the density.

By considering the torque exerted by the magnetic field, show that

$$\ell \equiv \kappa s^2\Omega - \frac{sB_\phi}{\mu_0}$$

is a function of χ only and hence that $\ell = \kappa s_A^2\omega$, where $s_A(\chi)$ is the value of s at the Alfvénic point. Provide a physical interpretation of this result and explain its significance.

3 Discuss the extent to which dynamo theory has so far succeeded in explaining the observed patterns of magnetic activity in stars like the sun.