UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS **Part III**

Thursday 7 June 2001 9 to 12

PAPER 14

BASIC ALGEBRAIC GEOMETRY

Answer any ***THREE*** *questions.*

Here k denotes an algebraically closed field of characteristic $\neq 2, 3$.

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 (a) Let $C, D \subset \mathbb{P}^2_k$ be curves of degrees n, m. Show that $|C \cap D| \leqslant n \cdot m$ unless C, D have a common component.

[*You may use standard properties of the resultant without proof.*]

(b) Let $C \subset \mathbb{P}^2_k$ be a reduced and irreducible curve of degree d with singular points $P_i \in C$ of multiplicities m_i. Show that $\sum m_i(m_i - 1) \leqslant (d-1)(d-2)$.

(c) Let $C \subset \mathbb{P}^2_k$ be a reduced and irreducible quartic with three double points.

Show that C is rational.

2 Let $C \subset \mathbb{P}^2_k$ be a nonsingular plane cubic. Show that

(a) C has an inflection point.

(b) C is described, in suitable coordinates

$$y^2 = x(x-1)(x-\lambda).$$

(c) C is not rational.

3 Let $S \subset \mathbb{P}^3_k$ be a nonsingular cubic surface. Assuming that S contains a line, show that:

(a) There are at most three lines through any point of S.

(b) Given a line $l \subset S$, there are five pairs $\{l_i, l'_i\}$ of lines on S such that:

(i) $l \cup l_i \cup l'_i$ is contained in a plane, for all i;

(ii) if $i \neq j$, $\{l_i \cup l'_i\} \cap \{l_j \cup l'_j\} = \phi$.

(c) S is birational to $\mathbb{P}^2_k$.

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Write an essay on dimension theory. Make sure you state the theorem on the dimension of fibres of a morphism, and you include the outline of some applications, such as the existence of a line on a nonsingular cubic surface.