UNIVERSITY OF CAMBRIDGE

MATHEMATICAL TRIPOS Part III

Friday 1 June 2001 9 to 11

PAPER 10

RAMSEY THEORY

*Attempt any **THREE** questions. The questions carry equal weight.*

You may not start to read the questions printed on the subsequent pages until instructed to do so by the Invigilator.

1 Let m be a positive integer. Show that, whenever $\mathbb{N}^{(2)}$ is red-blue coloured, there exists

either

(i) an m-term arithmetic progression M with $M^{(2)}$ blue

or

(ii) disjoint m-term arithmetic progressions A and B with every edge from A to B red.

[Hint: Suppose that every m-term arithmetic progression has at least one of its $\binom{m}{2}$ edges red. This gives a colouring of $\mathbb{N}^2$ with $\binom{m}{2}$ colours, by colouring $(a, d) \in \mathbb{N}^2$ according to which edge of the arithmetic progression with first term a and common difference d is red. Now apply Gallai's theorem.]

2 State and prove the Hales-Jewett theorem, and deduce van der Waerden's theorem. Prove the strengthened van der Waerden's theorem.

State Gallai's theorem, and show how to deduce it from the Hales-Jewett theorem.

3 State and prove Rado's theorem.

[You may assume that, for any m, p, c, whenever $\mathbb{N}$ is finitely coloured there is a monochromatic (m, p, c)-set.]

Deduce that, for any k, whenever $\mathbb{N}$ is finitely coloured there exist $x_1, \ldots, x_k$ with $FS(x_1, \ldots, x_k)$ monochromatic.

4 What does it mean to say that a subset of $\mathbb{N}^{(\omega)}$ is *Ramsey*? Prove that every $*$-Borel subset of $\mathbb{N}^{(\omega)}$ is Ramsey.