Part III Preparatory Workshops

Michaelmas Term 2018

The preparatory workshops are aimed at Part III students who have taken undergraduate courses in the relevant areas, but whose courses may not have covered all the material needed for the Part III lecture courses. We know that two hours cannot substitute for a whole undergraduate lecture course, so the workshops aim to give some intuition and a helping hand with further study.

Certain basics will be assumed in the workshops. If you find you are missing these also, please talk to someone about your course choices. Possible contacts are the Part III Course Directors, your college Director of Studies and the Part III Subject Advisors. The earlier you address any potential problems, the easier it will be for us to help you.

Below is the timetable for the workshops. Note that there are two sessions for each subject; in the first session you will be given some practice questions to attempt before the second session.

Subject	Date	Time	Room
Quantum Mechanics	4/10/18	2pm	MR2
	8/10/18	2pm	MR2
General Relativity	4/10/18	4pm	MR2
	8/10/18	4pm	MR2
Rings and Modules	4/10/18	2pm	MR3
	8/10/18	2pm	MR4
Algebraic Geometry	4/10/18	4pm	MR3
	8/10/18	4pm	MR4
Statistics	2/10/18	2pm	MR2
	9/10/18	2pm	MR5
Functional Analysis	2/10/18	2pm	MR14
	9/10/18	2pm	MR3
Measure Theory	2/10/18	3:30pm	MR14
	9/10/18	3:30pm	MR3
Fluids	4/10/18	3pm	MR9
	8/10/18	3pm	MR3
Algebraic Topology	2/10/18	5pm	MR14
	9/10/18	5pm	MR3

Quantum Mechanics

The topics we will touch are roughly

- Hilbert Space, Observables, etc.
- Dirac (Bra-Ket) Notation, Schrödinger & Heisenberg Picture
- Probabilities & Measurement
- Harmonic Oscillator, Spin Algebra, Ladder Operators
- Perturbation Theory Techniques
- ...

Furthermore, you are encouraged to bring any questions, requests or specific problems about QM to the lecture, so we can talk about those in detail.

We expect you to know

We expect you to have taken basic and advanced QM courses and be familiar with the concept of states (e.g. $|\psi\rangle$), operators (e.g. $\hat{x}$), time evolution (e.g. $\exp(it\hat{H})$), probabilities (e.g. $|\langle i|\psi\rangle|^2$) and the Schrödinger equation.

Useful for Part III courses

All Particle Physics, Quantum Fields and Strings courses, in particular Quantum Field Theory and Symmetries, Fields and Particles.

General Relativity

In this workshop we will cover the theory and techniques that are useful for all Part III Relativity and Gravitation courses but primarily the Michaelmas term course on General Relativity. We will cover:

- equivalence principles;
- structure of GR (qualitative);
- recap of Special Relativity;
- index gymnastics for GR;
- Euler-Lagrange equations and geodesics;
- manifolds.

The last four topics above will be covered in a modular fashion: there will be exercise sheets on all four of them and you will be able to choose which one to focus on.

We expect you to know

Special relativity; index notation and summation convention.

Useful for Part III courses

All Relativity and Gravitation courses but primarily General Relativity and Black Holes. Some topics (e.g. Special Relativity) are useful for the High Energy Physics courses, listed under Particle Physics, Quantum Fields and Strings.

Rings and Modules

This workshop will cover definitions useful for Part III algebra courses. We'll aim to give lots of examples, including fairly detailed looks at rings of polynomials and rings of algebraic numbers, which may help get experience for courses in algebraic geometry or algebraic number theory.

Session 1: Rings

We will try to cover:

- Ideals; maximal and prime ideals, and factorization
- Fields, principal ideal domains, unique factorization domains, integral domains
- Rings of algebraic numbers; polynomial rings
- Noetherian rings, Hilbert's basis theorem

There will be lots of exercises to give you practice at working with rings.

Session 2: Modules

We will aim to cover:

- Definition of a module; submodules, quotients, homomorphisms; compar- ison with vector spaces
- Finitely generated modules and Noetherian modules
- The structure theorem for finitely generated modules over a principal ideal domain; applications (to abelian groups and Jordan normal form).

We expect you to know

- Basic group theory: familiarity with abstract groups, homomorphisms, quotients, direct products.
- Basic linear algebra: vector spaces and related concepts, such as linear independence, spanning sets, and bases.
- The basic definitions of rings, ring homomorphisms and ideals - although I'll provide notes on this if you need a reminder.

Useful for Part III courses

Algebra, Lie algebras and their Representations, Iwasawa Algebras, Algebraic Geometry.

Algebraic Geometry

This workshop will review some classical algebraic geometry with the aim of providing some concepts and intuitions needed in the part III course. During the first session we will (attempt to) cover:

- Affine varieties (over an algebraically closed field)
- Zariski topology
- The Nullstellensatz
- Rings of regular and rational functions
- Morphisms
- Projective varieties

This is roughly the material contained within the preliminary section of the notes from a previous version of the course available at https://www.dpmms.cam.ac.uk/ pmhw/PtIIIAG2014.pdf

We expect you to know

- Familiarity with point set topology
- Familiarity with rings and ideals

The second session will be used to talk about exercises from the first, answer any questions and explain any other concepts from the first week of lectures if needed.

Useful for Part III courses

Algebraic Geometry.

Statistics

The first session will focus on applied statistics as taught in the undergraduate Statistical Modelling course. The core of this session will be results for the linear model (confidence regions, hypothesis tests, model selection) and, if there's time, some for the generalised linear model. The second session will focus on theoretical statistics as taught in the undergraduate Principles of Statistics course. In this session we will cover maximum likelihood estimators (MLEs) and their asymptotic properties. We will also briefly cover the Bayesian approach in this half. The main theorems covered (we will state but not prove these) will be:

- Gauss-Markov;
- Cochran's theorem;
- Wilks' theorem;
- asymptotic distribution of the MLE;
- Cramer-Rao.

We expect you to know

- Basic probability theory, including common distributions such as Binomial, Normal, Poisson, Chi-squared etc.
- Basic linear algebra (properties of orthogonal matrices).
- Some familiarity with basic statistical procedures (terminology like type I/II error).

Useful for Part III courses

Statistical Learning in Practice, Modern Statistical Methods, Astrostatistics, Statistics in Medical Practice, Analysis of Survival Data, Topics in Statistical Theory, Bayesian Modelling and Computation.

Functional Analysis

In this workshop we will cover concepts relevant to the to the courses on Partial Differential Equations and Ergodic Theory. We will focus on the main ideas and results in the following areas:

- Banach spaces, dual spaces and linear maps
- Baire Category Theorem and some consequences
- (Separable) Hilbert spaces, dual and adjoint maps
- $C(K)$ spaces
- Mollifiers and smooth functions

The workshop will be a mixture of lectures and exercises.

We expect you to know

Familiarity with linear algebra and basic analysis will be helpful.

Useful for Part III courses

Analysis of Partial Differential Equations, Elliptic Partial Differential Equations, Ergodic Theory.

Measure Theory

This course will cover the basics of measure theory that are required for most Part III courses in analysis. The topics covered will be:

- Construction of measures
- Lebesgue integration and Lebesgue spaces
- Integral convergence theorems
- Product measures and Fubini's theorem
- Independence and the Borel-Cantelli lemmas
- Convergence of random variables
- Important inequalities

We expect you to know

Familiarity with the definition of a measure and basic calculations with measures, integration on the real line and in Euclidean space, basic notions from probability such as random variables, expectation etc.

Useful for Part III courses

Analysis of Partial Differential Equations, Ergodic Theory, Elliptic Partial Differential Equations, Advanced Probability, Stochastic Calculus and Applications, Schramm-Loewner Evolutions.

Fluids

In this workshop we will review some basic ideas and work through some examples from the most important topics of Fluid Mechanics that will help you revise and cover potential gaps for Part III Continuum Mechanics courses. Two main methods that we will work with in the examples are vector calculus, scaling analysis and similarity solution.

Topics from which examples will be drawn:

- Basics of Fluid Mechanics
- Stokes flows
- Lubrication theory
- Vorticity
- Boundary layers
- Kevin- Helmholtz instability

We expect you to know

The Navier-Stokes Equations; Lagrangian and Eulerian co-ordinates; Convective derivative; Index (or Einstein) notation; Grad, Div and Curl; Stokes Theorem; The Divergence Theorem; Cylindrical and spherical polar co-ordinates.

Useful for Part III courses

Slow Viscous Flow, Fluid Dynamics of the Environment, Hydrodynamic Stability, Perturbation Methods, Fluid Dynamics of the Solid Earth, Fluid Dynamics of the Climate, Theoretical Physics of Soft Condensed Matter, Direct and Inverse Scattering of Waves, Active Biological Fluids.

Algebraic Topology

This workshop will introduce homology in algebraic topology with the aim of providing a gentle and intuitive introduction to the Part III courses in this area. The workshop will focus on intuition and techniques for computation. We will cover the following concepts and results:

- homotopic maps and homotopy equivalence of spaces;
- the intuition behind singular homology;
- computing with simplicial homology;
- chain complexes and exact sequences;
- the Mayer-Vietoris sequence and applications.

Few, if any, proofs will be given and some definitions will not be stated with full rigour. Where possible, I will instead illustrate concepts with examples and exercises.

We expect you to know

Standard undergraduate group theory, including the structure theorem for finitely generated abelian groups. Fluency with definitions in basic point-set topology, such as compactness and connectedness. Knowledge of the fundamental group would be good.

Useful for Part III courses

Algebraic Topology, Symplectic Topology, 3-Manifolds, Introduction to Geometric Group Theory.