Analysis of Functions (D) **16 lectures, Lent term**

Part II Linear Analysis and integration theory from Part II Probability and Measure are essential.

Lebesgue integration theory
Review of integration: simple functions, monotone and dominated convergence; existence of Lebesgue measure; definition of L^p spaces and their completeness. The Lebesgue differentiation theorem, Egorov's theorem, Lusin'stheorem. Mollification by convolution, continuity of translation and separability of L^p when $p \neq \infty$ [3]

Banach and Hilbert space analysis
Strong, weak and weak-* topologies. Review of the Riesz representation theorem for Hilbert spaces; reflexive spaces. Orthogonal systems of functions and their completeness. Hermite polynomials, the Haar basis. Compactness: review of the Ascoli–Arzelà theorem; weak-* compactnesss of the unit ball (both the separable and non-separable cases). The Riesz representation theorem for spaces of continuous functions. The Hahn–Banach theorem. Review of the Baire category theorem and its consequences: the open mapping theorem and the Banach–Steinhaus theorem. [5]

Fourier analysis
Definition of Fourier transform in L^1. Extension to L^2 by density and Plancherel's isometry. Fourier inversion theorem. Duality between regularity in real variable and decay in Fourier variable. Representation of L^2 periodic functions by Fourier series; the Poisson summation formula. Construction of solutions for linear PDEs with constant coefficients. [3]

Generalized derivatives and function spaces
Definition of generalized derivatives and of the basic spaces in the theory of distributions: $\mathcal{D}/\mathcal{D}'$ and $\mathcal{S}/\mathcal{S}'$. Definition of the Sobolev spaces H^k in $\mathbb{R}^d$ and the periodic d-cube. Sobolev embedding. The Rellich–Kondrashov theorem. The trace theorem. Construction and regularity of solutions for the Dirichlet problem of Laplace's equation. [5]

Appropriate books

H. Brézis *Functional Analysis, Sobolev Spaces and Partial Differential Equations.* Universitext, Springer 2011

A.N. Kolmogorov, S.V. Fomin *Elements of the Theory of Functions and Functional Analysis.* Dover Books on Mathematics 1999

E.H. Lieb and M. Loss *Analysis.* Second edition, AMS 2001