Analytic number theory studies the properties of integers using techniques from analysis, both real and complex. This course will give an introduction to the topic, focusing especially on the classical theory of the Riemann zeta function. A particular highlight is a proof of the Prime Number Theorem, which gives an asymptotic formula for the number of prime numbers, and which has an intimate relationship with the distribution of zeros of the zeta function. We will seek to understand the distribution of these zeros in some depth, including a discussion of the consequences of the infamous Riemann Hypothesis.

Topics will include:

- An introduction to Dirichlet series and the Riemann zeta function, including the functional equation and analytic continuation;
- Perron’s formula and the proof of the Prime Number Theorem;
- Quantitative zero-free regions for the zeta function;
- An explicit formula for the prime counting function in terms of zeros of the zeta function;
- Zero density results;
- The Riemann Hypothesis and its consequences.

Pre-requisites

Knowledge of basic complex analysis will be assumed, up to and including the maximum modulus principle and evaluating contour integrals using the method of residues. The more specialised analytic tools required will be developed in the course.

As this course focuses exclusively on the integers, knowledge of any algebraic number theory will not be required, and this course may be taken independently of any other number theory courses.

Literature


Additional support

Four examples sheets will be provided and four associated examples classes will be given, along with informal drop-in sessions. There will be a one-hour revision class in the Easter Term.