Infinite Games (L24)

Professor Benedikt Löwe

Infinite two-player perfect information games are connected to many topics in the foundations of mathematics: central concepts from analysis and topology can be reformulated in game-theoretic terms using infinite two-player perfect information games. Examples are the concepts of Lebesgue measurability, the property of Baire, as well as the perfect set property.

The central game-theoretic notion is the concept of *determinacy*: the full axiom of determinacy AD ("all infinite two-player perfect information games with natural number moves are determined") contradicts the axiom of choice AC, but definable fragments of AD can be proved in ZFC or extensions of ZFC. The axiom of determinacy itself yields an interesting alternative foundations of mathematics.

We shall treat several of the following topics:

Basic theory of determinacy. Applications in topology and measure theory. Incompatibility of AC and AD. Basics of descriptive set theory: the Borel hierarchy and the projective hierarchy.

Proving determinacy. Open determinacy. Low-level Borel games. Borel determinacy.

Proving determinacy from large cardinals. Introduction to large cardinals: inaccessible cardinals and measurable cardinals. Proving $\mathbf{\Pi}^1_1$-determinacy from a measurable cardinal.

The axiom of determinacy. Combinatorial consequences: $\aleph_1$ and $\aleph_2$ are measurable. Infinite exponent partition relations.

Stronger axioms of determinacy. The axiom of real determinacy. Inconsistent extensions of the axiom of determinacy. Long games.

The Wadge hierarchy. Definition and structure theory of the Wadge hierarchy under AD.

A course webpage will be available at

https://www.math.uni-hamburg.de/home/loewe/Lent2020/.

Pre-requisites

The Part II course *Logic and Set Theory* or an equivalent course is essential. The Part Ib course *Metric and Topological Spaces* is useful.

Literature

1. Akihiro Kanamori, *The Higher Infinite. Large Cardinals in Set Theory from Their Beginnings.* Springer 2003 [Springer Monographs in Mathematics]

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.