

Symplectic Topology (L24)

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The study of symplectic manifolds originated as an extension of classical mechanics; it has since developed into a field of its own right, with connections to e.g. low-dimensional topology, algebraic geometry, and theoretical physics. The course will focus on the core foundations of symplectic topology, with an emphasis on explicit geometric techniques and examples, and end with an introduction to J -holomorphic curves, which are at the heart of modern symplectic topology.

Time allowing, topics are expected to include:

- Symplectic linear algebra. Hamilton's equations, cotangent bundles. Lagrangian submanifolds. Symplectic submanifolds. Moser's trick, Darboux and Weinstein neighbourhood theorems.
- Symplectic circle actions and moment maps, symplectic reduction.
- Surgery constructions: blow ups, symplectic fibre sums. Lefschetz pencils. Gompf's theorem on fundamental groups of symplectic 4-manifolds.
- Almost complex structures and compatible triples. Some properties of Kaehler manifolds.
- Basic properties of pseudo-holomorphic curves. Gromov non-squeezing. Symplectic capacities, Eliashberg's theorem on C^0 -closure of $Symp$ in $Diff$.

Pre-requisites

Essential: Algebraic Topology and Differential Geometry, at the level of the Part III Michaelmas courses. Basic concepts from Algebraic Geometry (at the level of the Part II course) will be useful.

Literature

1. D. McDuff and D. Salamon, *Introduction to symplectic topology*, 3rd edition. Oxford University Press, 2017.
2. D. McDuff and D. Salamon, *J-holomorphic curves and symplectic topology*, 2nd edition. American Mathematical Society, 2012.
3. A. Cannas da Silva, *Lectures on symplectic geometry*, Springer-Verlag, 2001.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.