Random groups and related topics (L16)

Non-examinable (Graduate Level)

One classical way to define a group is via a presentation by generators and relations. Putting a probability measure on the set of presentations, one can make sense of the notion of random group and start asking questions about the properties of a typical group, or make use of the "probabilistic method" to construct groups with interesting properties. Of course various probabilistic models arise naturally, but by far the most studied in the last twenty years is the density model of Misha Gromov. This will be the main focus of the course. I will touch upon a variety of topics as random groups will serve as a pretext to discuss other important notions in modern infinite group theory, such as small cancellation theory, word hyperbolic groups, Burnside groups, Kazhdan's property (T), amenability, growth, expander graphs, random walks on groups, etc.

Pre-requisites

This course has no genuine prerequistes but assumes some familiarity with undergraduate topology, geometry, group theory and probability. It may be taken in parallel with the Part III course on Geometric Group Theory.

Literature

1. C. Drutu and M. Kapovich, *Geometric Group Theory*, Colloquium Publications 63, (2018).
2. M. Gromov, *Asymptotic invariants of infinite groups*, in Geometric group theory, ed. G. Niblo, M. Roller, Cambridge University Press, Cambridge (1993).
3. M. Gromov, *Random walk in random groups*, GAFA, Geom. Funct. Anal. 13 (2003), no. 1, 73–146.
4. Y. Ollivier, *A January 2005 invitation to random groups*, Ensaios Matemáticos 10. Sociedade Brasileira de Matemática, (2005) +100 pp.
5. B. Bekka, P. de la Harpe, A. Valette, *Kazhdan's property (T)*, New Mathematical Monographs, 11. Cambridge University Press, Cambridge, (2008).

Additional support

One or two Part III essays will be offered in connection to some of the topics of the course.