Geometric Inverse Problems (L24)

Non-Examinable (Part III Level)

Gabriel P. Paternain

The study of geometric inverse problems is typically motivated by inverse problems in PDEs, geophysics and medical imaging. The main goal is the reconstruction of geometric structures (metrics, connections, vector bundles etc.) from either boundary measurements or local measurements. The course will describe recent developments in the area with an emphasis on the 2D picture.

The first part of the course will include a thorough discussion of the geodesic X-ray transform for functions and tensors when the background manifold is simple (i.e. it has strictly convex boundary, no conjugate points and is simply connected). Then we shall move on to attenuated X-ray transforms, including versions for connections and Higgs fields (systems).

The second part of the course will address non-linear geometric inverse problems. A full proof of boundary rigidity for simple surfaces will be given along with proofs for the recovery of a connection and a Higgs field from scattering data.

While a geometric background will be desirable, it is not strictly necessary, as the relevant tools in 2D can be provided on demand.

Pre-requisites

Part III (or Part II) Differential Geometry and Analysis of Partial Differential Equations will be helpful.

Preliminary Reading

1. P. Kuchment, *The Radon transform and medical imaging.* CBMS-NSF Regional Conference Series in Applied Mathematics, **85**. Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 2014.

Literature

1. J. Ilmavirta and F. Monard, *Integral geometry on manifolds with boundary and applications.* Available at

   https://arxiv.org/pdf/1806.06088.pdf

2. L. Pestov and G. Uhlmann, *Two dimensional compact simple Riemannian manifolds are boundary distance rigid.* Ann. of Math. **161** (2005) 1093–1110.

3. G.P. Paternain, M. Salo and G. Uhlmann, *The attenuated ray transform for connections and Higgs fields.* Geom. Funct. Anal. **22** (2012) 1460–1489.

4. *Lecture Notes.* In preparation; to be given during the course.

Additional support

The lecture notes will contain plenty of exercises and example classes can be set up if there is demand, bearing in mind the non-examinable nature of the course.