Turán’s theorem, giving the maximum size of a graph that contains no complete \( r \)-vertex subgraph, is an example of an extremal graph theorem. Extremal graph theory is an umbrella title for the study of how graph and hypergraph properties depend on the values of parameters. This course builds on the material introduced in the Part II Graph Theory course, which includes Turán’s theorem and an introduction to the Erdős-Stone theorem.

The first half of the course will discuss the fundamental structural results of the subject, namely the Erdős-Stone theorem and Szemerédi’s Regularity Lemma, together with some of their consequences. The second half of the course will consider the application of simple probabilistic arguments to the solution of extremal problems. It will include a discussion of the semi-random method (showing, for example, the existence of almost perfect Steiner systems).

Pre-requisites

An awareness of the basic concepts, techniques and results of graph theory, as afforded by the Part II Graph Theory course (such as Turán’s theorem, Ramsey’s theorem, Hall’s theorem and so on) will be assumed, though not relied upon. Familiarity with the most elementary aspects of Probability Theory (such as the inequalities of Markov and Chebychev) will also be useful.

Literature

No book covers the course but the following can be helpful.


Additional support

Example sheets will be supplied, and three hours of examples classes will be given. There will be a further revision class in the Easter Term.