Approximate group actions and Ulam stability (L16)

Non-Examinable (Part III Level)

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The following is an example of a problem in group theoretic Ulam stability: Given two permutations $A, B \in \mathrm{Sym}(n)$ such that AB and BA are almost equal, can we always find $A', B' \in \mathrm{Sym}(n)$ such that A' is close to A, B' is close to B, and $A'B' = B'A'$? Equivalently, given an approximate action of $\mathbb{Z}^2$ on a finite set, is it necessarily close to a genuine action?

This question was formulated and given an affirmative answer by Arzhantseva and Paunescu. Accordingly, we say that $\mathbb{Z}^2$ is *stable in symmetric groups*, or *stable* for short. The topic of this course is the more general question: Which finitely generated groups are stable?

I aim to cover the following topics:

- The classification of amenable stable groups in terms of invariant random subgroups;
- Stability of surface groups;
- Stability and property (T);
- Relations to sofic groups and (non-)approximability problems;
- Applications to property testing and computer science.

Pre-requisites

Most of the topics require only basic knowledge in group theory. Familiarity with basic notions from geometric group theory would be useful.

Literature

1. A. Thom. *Finitary approximations of groups and their applications.* Proceedings of the ICM 2018.

2. M. De Chiffre, L. Glebsky, A. Lubotzky, A. Thom. *Stability, cohomology vanishing, and non-approximable groups.* Available at

   https://arxiv.org/abs/1711.10238

3. N. Lazarovich, A. Levit, Y. Minsky. *Surface groups are flexibly stable*

   https://arxiv.org/abs/1901.07182

4. G. Elek, *Finite graphs and amenability*, J. Funct. Anal. 263 (9) (2012) 2593–2614.

5. O. Becker, A. Lubotzky, A. Thom. *Stability and Invariant Random Subgroups*, Duke Math. J. (to appear). Also available at

   https://arxiv.org/abs/1801.08381