Algebra(M24)

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The primary aim of the course is to give an introduction to the theory of commutative Noetherian algebras and modules, a theory that is an essential ingredient in algebraic geometry, algebraic number theory and representation theory. In particular we shall learn about (commutative) polynomial and power series algebras.

I shall also include a small amount of introductory material about non-commutative algebras.

Topics I hope to fit in will be

Examples, tensor products. Ideal theory for commutative Noetherian algebras, localisations. Artinian algebras (commutative and non-commutative), Artin-Wedderburn theorem. Integral dependence. Dimension theory. Filtrations and associated graded algebras. Injective and Projective modules; Ext and Tor. Derivations and differential operators. Hochschild (co-)homology.

Pre-requisites

It will be assumed that you have attended a first course on ring theory, eg IB Groups, Rings and Modules. Experience of other algebraic courses such as II Representation Theory, Galois Theory or Number Fields will be helpful but not necessary.

Literature

1. M.F. Atiyah and I.G. Macdonald, Introduction to commutative algebra, Addison-Wesley, 1969.

2. N. Bourbaki, Commutative algebra, Elements of Mathematics, Springer, 1989.

3. I. Kaplansky, Commutative rings, University of Chicago Press, 1974.

4. H. Matsumura, Commutative ring theory, Cambridge Studies 8, Cambridge University Press, 1989.

5. M.Reid, Undergraduate Commutative Algebra, LMS student texts 29, Cambridge University Press, 1995.

6. R.Y. Sharp, Steps in commutative algebra, LMS Student Texts 19, Cambridge University Press, 1990.

   The basic introductory text for commutative algebra is Atiyah and Macdonald but it doesn't go into much detail and many results are left to the exercises. Sharp fills in some of the detail but neither book goes far enough. Both Kaplansky and Matsumura cover additional material though Matsumura is a bit tough as an introduction. Reid's book is a companion to one on algebraic geometry and that influences his choice of topics and examples. Bourbaki is encyclopaedic.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.