Logic

The following multiple choice self-test questions are intended to give an indication whether you have prerequisites for the Part III courses in logic. All of them are basic and easy questions in logic: you should get most of them right without effort or preparation. [Last updated: 19 August 2024.]

1. Let $\mathcal{L}$ be a countable language and $\mathrm{S}^{\mathcal{L}}$ the set of $\mathcal{L}$-sentences. Only one of the following statements is true. Which one?

   ☐ **A** The set $\mathrm{S}^{\mathcal{L}}$ is always uncountable.

   ☐ **B** The set $\mathrm{S}^{\mathcal{L}}$ is always countably infinite.

   ☐ **C** The set $\mathrm{S}^{\mathcal{L}}$ is always countable, but can be finite for some choices of $\mathcal{L}$.

   ☐ **D** The set $\mathrm{S}^{\mathcal{L}}$ can be countable or uncountable, depending on the choice of $\mathcal{L}$.

2. Let $\mathcal{L}$ be a countable language, Φ be a set of $\mathcal{L}$-sentences, and $\mathfrak{T}^{\Phi}$ be the term model of Φ (as constructed in Henkin's proof of the completeness theorem). Only one of the following statements is true. Which one?

   ☐ **A** It is always the case that $\mathfrak{T}^{\Phi} \models \Phi$.

   ☐ **B** The set Φ is consistent if and only if $\mathfrak{T}^{\Phi} \models \Phi$.

   ☐ **C** If Φ is *negation-complete* (i.e., for all φ, we have that either $\Phi \models \varphi$ or $\Phi \models \neg\varphi$), then $\mathfrak{T}^{\Phi} \models \Phi$.

   ☐ **D** None of the above.

3. Let $\mathcal{L}$ be any language and Φ be a set of $\mathcal{L}$-sentences. Of the following four statements, only one cannot be true of Φ. Which one?

   ☐ **A** The set Φ has no infinite models.

   ☐ **B** The set Φ has finite models of arbitrarily large size.

   ☐ **C** If $\mathfrak{M}$ is a model of Φ, then the cardinality of the universe of $\mathfrak{M}$ is a square number (i.e., n^2 for some $n \in \mathbb{N}$).

   ☐ **D** The set Φ has no finite models.

4. Let $\mathcal{L}$ be the language with a single binary function symbol $\times$ and consider the rational numbers with multiplication $(\mathbb{Q}, \cdot)$ as an $\mathcal{L}$-structure. Only one of the following sets is not $\mathcal{L}$-definable in $(\mathbb{Q}, \cdot)$. Which one?

   ☐ **A** $\{-2\}$.

   ☐ **B** $\{-1\}$.

   ☐ **C** $\{0\}$.

   ☐ **D** $\{1\}$.

5. Let $\mathcal{L}$ be a language and κ be a cardinal. A set Φ of $\mathcal{L}$-sentences is called *finitely categorical* if any two finite models of the same cardinality are isomorphic; it is called *countably categorical* if any two countable models of the same cardinality are isomorphic; it is called *uncountably categorical* if any two uncountable models of the same cardinality are isomorphic. Exactly one of the following statements is true. Which one?

☐ **A** Every finitely categorical theory is countably categorical.
☐ **B** Every countably categorical theory is uncountably categorical.
☐ **C** Every uncountably categorical theory cannot have finite models.
☐ **D** None of the above.

6. For structures $\mathfrak{M}$ and $\mathfrak{N}$, we write $\mathfrak{M} \cong \mathfrak{N}$ for "$\mathfrak{M}$ and $\mathfrak{N}$ are isomorphic", $\mathfrak{M} \equiv \mathfrak{N}$ for "$\mathfrak{M}$ and $\mathfrak{N}$ are elementarily equivalent", and $\mathfrak{M} \preccurlyeq \mathfrak{N}$ for "$\mathfrak{M}$ elementarily embeds into $\mathfrak{N}$". One of the following statements is false. Which one?

☐ **A** If $\mathfrak{M} \cong \mathfrak{N}$, then $\mathfrak{M} \equiv \mathfrak{N}$.
☐ **B** If $\mathfrak{M} \equiv \mathfrak{N}$, then $\mathfrak{M} \preccurlyeq \mathfrak{N}$.
☐ **C** If $\mathfrak{M} \preccurlyeq \mathfrak{N}$, then $\mathfrak{M} \equiv \mathfrak{N}$.
☐ **D** If $\mathfrak{M} \cong \mathfrak{N}$, then $\mathfrak{M} \preccurlyeq \mathfrak{N}$.

7. Let **C** be the class of all infinite groups. One of the following statements is true. Which one?

☐ **A** The class **C** is not axiomatisable in the language of groups.
☐ **B** The class **C** is axiomatisable in the language of groups, but not finitely axiomatisable.
☐ **C** The class **C** is finitely axiomatisable in the language of groups.
☐ **D** Whether the class **C** is axiomatisable or not is independent of the axioms of set theory.

8. Only one of the following classes of structures is axiomatisable in the natural language relevant for it. Which one?

☐ **A** The class of finite rings.
☐ **B** The class of countable fields.
☐ **C** The class of uncountable graphs.
☐ **D** The class of groups isomorphic to a subgroup of the monster group.

9. Consider the language with one binary relation symbol R and the set Φ containing the axioms

$$\begin{array}{rl}
\varphi_{\mathrm{R}} & \forall x(x\ R\ x), \\
\varphi_{\mathrm{S}} & \forall x \forall y(x\ R\ y \to y\ R\ x), \\
\varphi_{\mathrm{T}} & \forall x \forall y \forall z((x\ R\ y \wedge y\ R\ z) \to x\ R\ z), \\
\varphi_{\leq 2} & \forall x \forall y \forall z(x\ R\ y \vee y\ R\ z \vee x\ R\ z), \\
\varphi_{\geq 2} & \exists x \exists y(\neg x\ R\ y), \text{ and} \\
\varphi_n & \forall x \exists y_1 \ldots \exists y_n(\bigwedge\limits_{\substack{1 \leq k, \ell \leq n \\ k \neq \ell}} y_k \neq y_\ell \wedge \bigwedge\limits_{1 \leq k \leq n} x \neq y_k \wedge x\ R\ y_k), \text{ for all } n.
\end{array}$$

Only one of the following statements is true. Which one?

☐ **A** The set Φ is inconsistent.
☐ **B** The set Φ has finite models.
☐ **C** The set Φ has at least two non-isomorphic models of the same size.
☐ **D** All models of Φ are countably infinite.