# Random planar geometry (L16)

**Jason Miller**

This course will be an introduction to two-dimensional random geometric structures, both discrete and continuous.

The first part of the course will be on *random planar graphs*. Recall that a *tree* is a connected graph without cycles. A *plane tree* is a tree together with an embedding into the plane. Since there are only a finite number of plane trees with a fixed number of edges, one can pick one uniformly at random. We will discuss how random plane trees are related to random walks on $\mathbf{Z}$. We will also describe their continuous counterpart, the so-called *continuum random tree*, which is a random tree defined using Brownian motion. Plane trees turn out to serve as the basic building block for more elaborate geometric structures. One important example is a *planar map*, which is a graph together with an embedding into the plane so that no two edges cross. Since there are only a finite number of planar maps with a fixed number of faces, one can also talk about picking one uniformly at random. This is an example of a *random planar map*. The study of random planar maps goes back to work of Tutte in the 1960s in his attempt to prove the four color theorem. In recent years, random planar maps have been the subject of intense study, in part due to their deep connection with understanding different models in statistical mechanics (e.g., the percolation model and loop-erased random walk).

The second part of the course will be on the Schramm-Loewner evolution (SLE), which is a family of non-crossing curves in the plane indexed by a parameter $\kappa \geq 0$. SLE was introduced by Schramm in 1999 to describe the scaling limit of many different models in statistical mechanics (e.g., the percolation model and loop-erased random walk) in the same way that Brownian motion describes the scaling limit of simple random walk. SLE is defined in a very interesting way, combining ideas from complex analysis and probability. In this part of the course, we will introduce SLE and derive some of its basic properties.

Time permitting, we will discuss more advanced topics, such as the Gaussian free field and its connection with random planar maps and SLE.

### Pre-requisites

Advanced probability. Stochastic calculus is a co-requisite.

### Literature

1. G. Miermont *Aspects of random maps.* 2014 St Flour lecture notes.
   http://perso.ens-lyon.fr/gregory.miermont/coursSaint-Flour.pdf
2. J. Miller *Schramm-Loewner evolutions.*
   http://statslab.cam.ac.uk/~jpm205/teaching/lent2019/sle_notes.pdf
3. N. Berestycki and J.R. Norris, *Lectures on Schramm-Loewner evolutions.*
   http://www.statslab.cam.ac.uk/~james/Lectures/sle.pdf

### Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a revision class in the Easter Term.