Information Theory (L16)

Ioannis Kontoyiannis

Information theory is the mathematical foundation of the ideas and tools required for the quantitative description and analysis of the notion of information. The origin of the theory in Claude Shannon's landmark 1948 paper is motivated by questions in communications engineering, but since then information theory has forged deep connections with many areas of mathematics, most notably with probability and statistics. This will be an introductory course to the main information-theoretic ideas, results, and techniques. We will discuss entropy as a measure of information, relative entropy as a natural distance between probability distributions, and mutual information as a universal dependence measure between random variables. Their properties will be established (monotonicity, chain rules, data processing inequalities, asymptotic equipartition) and Shannon's two main theorems will be proved: The source coding theorem that establishes the entropy as the fundamental limit for data compression, and the channel coding theorem which identifies the capacity as the fastest possible rate of reliable data transmission through a noisy channel.

Pre-requisites

The only pre-requisite is knowledge of basic probability, although a certain level of maturity and familiarity with the use of probabilistic techniques will be helpful. Knowledge of advanced (including measure-theoretic) probability is not necessary.

Literature

1. Mainstream introduction to information theory:
   T.M. Cover and J. Thomas. *Elements of Information Theory.* 2nd edition. Wiley-Interscience, 2006.

2. A more applied perspective:
   D. MacKay. *Information Theory, Inference, and Learning Algorithms.* Cambridge University Press, 2003. Available free online at: https://www.inference.org.uk/itprnn/book.pdf.

3. Theoretical connections with ergodic theory and probability...
   P. Billingsley. *Ergodic Theory and Information.* J. Wiley, 1965.

4. ... and: P. Shields. *The Ergodic Theory of Discrete Sample Paths.* American Mathematical Society, 1996.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a revision class in the Easter Term.