Local Fields (M24)

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The p-adic numbers $\mathbb{Q}_p$ were introduced by Hensel at the end of the 19th century and are now a ubiquitous tool in modern number theory as well as many other fields including algebraic topology, representation theory and algebraic geometry. The idea is to consider the completion of $\mathbb{Q}$ with respect to the absolute value defined by $|x|_p = p^{-n}$ for non-zero $x \in \mathbb{Q}$ where $x = p^n \frac{a}{b}$ with $a, b, n \in \mathbb{Z}$ and a, b coprime to p. The resulting field $\mathbb{Q}_p$ gives a neat way of packaging the information of congruences modulo n for all n and is the basic example of a local field. From this point of view, local fields are objects lying on the interface between algebra and analysis and the techniques used to study them involve an interesting mix of the two subjects.

This course covers the basic theory of local fields and is likely to be useful for students interested in studying other Part III courses on number theory such as Elliptic Curves. Topics to be covered will include:

Absolute values on fields; valuations; structure of local fields;

Extensions of complete fields; Galois theory; the different and discriminants;

Decomposition groups; Hensel's lemma; ramification theory;

Local class field theory (statements only).

Pre-requisites

Basic algebra up to and including Part II Galois theory as well as knowledge of concepts in point set topology and metric spaces are essential pre-requisites. It will be assumed that students have attended a first course in algebraic number fields.

Literature

1. J.W.S. Cassels, *Local fields*, Cambridge University Press, 1986.

2. J. Neukirch, *Algebraic number theory*, Springer–Verlag, 1999.

3. J. P. Serre, *A course in arithmetic*, Graduate Texts in Mathematics, 7. Springer–Verlag, 1973.

4. J. P. Serre, *Local fields*, Graduate Texts in Mathematics, 67. Springer–Verlag, 1979.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.