Algebraic Number Theory (L24)

Professor A J Scholl

In recent decades one of the most growing areas of research in number theory has been Arithmetic Algebraic Geometry, in which the techniques of algebraic number theory and abstract algebraic geometry are applied to solve a wide range of deep number-theoretic problems. These include the celebrated proof of Fermat's Last Theorem, work on the Birch-Swinnerton-Dyer conjectures, the Langlands Programme and the study of special values of L-functions. In this course we will study one half of the picture: Algebraic Number Theory. I will assume some familiarity with the basic ideas of number fields, although these will be reviewed briefly at the beginning of the course.

Topics likely to be covered (not in order):

- Decomposition of primes in extensions, decomposition and inertia groups. Discriminant and different.
- Completion, adeles and ideles, the idele class group. Application to class group and units.
- Dedekind zeta function, analytic class number formula.
- Class field theory (statements and applications). L-functions.

Pre-requisites

Basic algebra up to and including Galois theory is essential. Familiarity with the Michaelmas term *Local Fields* course (or equivalent) will be assumed.

Exposure to number fields (at the level of the Part II course) is highly desirable.

Literature

1. J.W.S. Cassels and A. Frohlich: *Algebraic Number Theory.* London Mathematical Society 2010 (2nd ed.)
2. A. Frohlich, M.J. Taylor: *Algebraic Number Theory.* Cambridge, 1993.
3. J. Neukirch, *Algebraic number theory.* Springer, 1999.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.