Model Theory (M24)

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Model theory is a branch of mathematical logic in which one studies classes of models of first-order theories. In the early 20th century, a driving question was the extent to which a mathematical structure can be characterized by its first-order properties. The discovery and application of the Compactness Theorem marked a fundamental failure in this pursuit, since it showed that, no matter how rich a language one chooses, any infinite first-order structure admits elementarily equivalent structures of arbitrarily large cardinality. This led to the study of categoricity and classification of models of first-order theories, which came to fruition in the 1960s and 70s. More recent applications of model theory to other areas of mathematics have shown that this consequence of the Compactness Theorem, which may have initially been viewed as a setback, is in fact an extremely powerful tool. Indeed, one can gain deep insights into a mathematical structure by working instead with an elementarily equivalent structure satisfying various desirable properties, which provide leverage over what may have initially appeared to be intricate or mysterious behavior in the original structure.

This course will cover a selection of topics from model theory, such as:

- complete theories and axiomatizations of structures,
- quantifier elimination and applications,
- type spaces and saturated models,
- the Omitting Types Theorem and applications,
- categoricity and the number of countable models,
- an introduction to stability theory.

Pre-requisites

Logic and Set Theory (part II); or equivalent. I assume familiarity with first-order languages and structures, the Compactness Theorem, and the Lowenheim-Skolem Theorems.

Literature

1. D. Marker *Model Theory: An Introduction.* Springer, 2002.

Additional support

Three examples sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.