Theoretical Physics of Soft Condensed Matter (L24)

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Soft Condensed Matter refers to liquid crystals, emulsions, molten polymers and other microstructured fluids or semi-solid materials. Alongside many high-tech examples, domestic and biological instances include mayonnaise, toothpaste, engine oil, shaving cream, and the lubricant that stops our joints scraping together. Their behaviour is classical ($\hbar = 0$) but rarely is it deterministic: thermal noise is generally important.

The basic modelling approach therefore involves continuous classical field theories, generally with noise so that the equations of motion are stochastic PDEs. The form of these equations is helpfully constrained by the requirement that the Boltzmann distribution is regained in the steady state (when this indeed holds, i.e. for systems in contact with a heat bath but not subject to forcing). Both the dynamical and steady-state behaviours have a natural expression in terms of path integrals, defined as weighted sums of trajectories (for dynamics) or configurations (for steady state). These concepts will be introduced in a relatively informal way, focusing on how they can be used for actual calculations.

In many cases mean-field treatments are sufficient, simplifying matters considerably. But we will also meet examples such as the phase transition from an isotropic fluid to a 'smectic liquid crystal' (a layered state which is periodic, with solid-like order, in one direction but can flow freely in the other two). Here mean-field theory gets the wrong answer for the order of the transition, but the right one is found in a self-consistent treatment that lies one step beyond mean-field (and several steps short of the renormalization group, whose application to classical field theories is discussed in other courses but not this one).

Important models of soft matter include diffusive ϕ^4 field theory ('Model B'), and the noisy Navier-Stokes equation which describes fluid mechanics at colloidal scales, where the noise term is responsible for Brownian motion of suspended particles in a fluid. Coupling these together creates 'Model H', a theory that describes the physics of fluid-fluid mixtures (that is, emulsions). We will explore Model B, and then Model H, in some depth. We will also explore the continuum theory of nematic liquid crystals, which spontaneously break rotational but not translational symmetry, focusing on topological defects and their associated mathematical structure such as homotopy classes.

A section of the course will present the mechanical equations for low-dimensional soft materials informed by concepts of topology and differential geometry. We will first identify kinematic variables suitable for the description of one-dimensional materials like filaments and two-dimensional materials like membranes, and then consider dynamical conservation laws, emphasising their topological character. We will move on to material-specific relations that close the conservation laws, emphasising their geometric character. These general principles will be illustrated by examples of specific one- and two-dimensional materials.

Finally, the course will analyse soft-matter systems whose microscopic dynamics does not have time-reversal symmetry, such as self-propelled colloidal swimmers. We will discuss how the absence of time-reversal symmetry leads to qualitative changes in dynamical behaviour, both for averaged quantities and for fluctuations. This part of the course will describe some general results, particularly fluctuation theorems, and their consequences for the observation of rare events. The implications of these results in specific soft-matter systems may also be discussed.

Note on lectures

Approximately 16 lectures will be given by Prof. Cates followed by about 4 lectures each from Dr Adhikari and Dr Jack respectively.

Pre-requisites

Knowledge of Statistical Mechanics at an undergraduate level is essential. This course complements in part the following Michaelmas Term courses although none are prerequisites: Statistical Field Theory; Non-Newtonian Fluid Dynamics; Slow Viscous Flow; Quantum Field Theory.

In previous years the audience has included a mix of students whose main specialism is either fluid dynamics or field theory. People with these differing backgrounds may find different parts of the course easier or harder, but the intention is to create a roughly level playing field.

Preliminary Reading

1. D. Tong *Lectures on Statistical Physics*

   `http://www.damtp.cam.ac.uk/user/tong/statphys.html`

   Before embarking on this course you do need to understand the equation $F = -k_B T \ln Z$ and its implications. This includes knowing what the Boltzmann distribution is, what it describes, and when it is true. You should also have met the concept of chemical potential and the grand canonical ensemble. Familiarity with the Landau theory of phase transitions is desirable. We will not need much abstract thermodynamics (e.g. Maxwell relations) but you do need to know the zeroth, first and second laws. The above lecture notes are an excellent resource for revising and reviewing the key material.

2. M. E. Cates and E. Tjhung *Theories of binary fluid mixtures: from phase-separation kinetics to active emulsions.* J. Fluid Mech. (2018), **836**, pp1-66.

   `https://www.cambridge.org/core/journals/journal-of-fluid-mechanics/article/theories-of-binary-fluid-mixtures-from-phaseseparation-kinetics-to-active-emulsions/5BD133CB20D89F47E724D77C296FEF80/share/106fd30f307db12134745de39fd568fbbaa3f9d2`

   This JFM perspectives article has significant overlap (perhaps 40%) with the course but takes fluid mechanics as its starting point whereas we will start from statistical physics and bring in fluid mechanics when needed. It gives a good flavour of the types of problem we will address and some of the methodologies involved. However we will not have time to cover much of the material it contains on active systems.

Literature

So far there are no books that treat this material at the right level. But it may be worth looking at:

1. P. Chaikin and T. C. Lubensky *Principles of Condensed Matter Physics.* Cambridge University Press, 1995. An authoritative and broad ranging but advanced book, that is

worth dipping into to see how hydrodynamics, broken symmetries, topological defects all feature in the description of condensed matter systems at $\hbar = 0$. More for inspiration than information though; this course may help you in understanding the book, but probably not vice versa.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will also be a revision class in the Easter Term.

Unofficial lecture notes

Unofficial lecture notes were taken several years ago when this course was 16 lectures long and lectured by Prof. Cates alone – effectively the first 16 lectures of this course:

`https://dec41.user.srcf.net/h/III_L/theoretical_physics_of_soft_condensed_matter`

These notes are far from perfect but are mentioned here so that all students are equally aware of their existence.