

# Higher-order uniformity and applications (E12)

*Non-Examinable (Graduate Level)*

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The discrete Fourier transform is a useful, classical tool that allows us to measure the extent to which a set of integers is uniformly distributed. Higher-order generalisations of the Fourier transform provide a more sensitive measure of uniform distribution, and first appeared in work of Gowers on Szemerédi's theorem in the late 1990s. They were further developed by Green and Tao (amongst others), culminating in their celebrated proof that the primes contain arbitrarily long arithmetic progressions.

In this course we shall examine the foundations of higher-order Fourier decompositions, the analogous regularity results for hypergraphs (generalising Szemerédi's famous graph regularity lemma from the 1970s), and some of their applications. In addition to the Green-Tao theorem, we shall cover recent progress of Peluse on polynomial patterns in dense subsets of the integers.

If time permits, we shall give an outline of the more abstract framework for higher-order Fourier analysis developed over the past decade by Szegedy and others.

## Pre-requisites

This course assumes familiarity with the discrete Fourier transform and Szemerédi's regularity lemma for graphs. The Part III course *Additive Combinatorics* offered in the Lent term will be useful.

## Literature

1. B.J. Green, *Montréal lecture notes on quadratic Fourier analysis*. Additive Combinatorics (Montréal 2006, ed. Granville et al.), CRM Proceedings vol. 43, AMS (2007), 69-102. Available at <https://arxiv.org/abs/math/0604089>.
2. W.T. Gowers, *Hypergraph regularity and the multidimensional Szemerédi theorem*. Annals of Mathematics, 166 (2007), 897–946. Available at <https://arxiv.org/abs/0710.3032>.
3. J. Fox, D. Conlon, Y. Zhao, *The Green-Tao theorem: an exposition*. EMS Surv. Math. Sci. 1 (2014), 249-282. Available at <https://arxiv.org/abs/1403.2957>.
4. S. Peluse, *On the polynomial Szemerédi theorem in finite fields*, Duke Math. J. 168, no. 5 (2019), 749-774. Available at <https://arxiv.org/abs/1802.02200>.

## Additional support

Two examples sheets will be provided and associated office hours will be offered.