Additive Combinatorics (L24)

Thomas Bloom

Additive combinatorics lies in the intersection of number theory and combinatorics, and studies the generic behaviour of simple arithmetic operations, such as addition and multiplication, on finite sets. For example, under what conditions can we guarantee that a set contains a three-term arithmetic progression? How does the size of the sumset $A + A$ differ from the size of A? How do addition and multiplication interact in sets of integers? It differs from other branches of combinatorics in that it introduces basic algebraic operations, and differs from other branches of number theory in that it rarely assumes much about the sets we are working in, other than very basic information such as the size of the set.

Despite the basic nature of the objects it studies, additive combinatorics as a field is relatively recent, and there have been many exciting recent developments. In this course our goal is to build up the basic theory of the subject from scratch, including some recent new techniques, so that we can discuss and prove some of the great theorems of the area, and engage with the latest research.

Topics that will be covered include

- Basic sumset estimates (such as the Balog-Szemerédi-Gowers lemma and the Plünnecke-Petridis bound),
- Fourier analysis in finite abelian groups, and how this can be used to prove Roth's theorem on three-term arithmetic progressions,
- Almost-periodicity, a probabilistic sampling technique,
- Long arithmetic progressions in sumsets, and
- The Freiman-Ruzsa inverse theorem and its quantitative improvements by Sanders.

Anyone who enjoys either combinatorics and/or number theory should find something of interest in this course.

Pre-requisites

There are no pre-requisites for this course.

Literature

Parts of this course are quite recent, and not yet present in any textbook. Complete PDF lecture notes will be provided by the lecturer. Some of the topics covered are discussed in the following books.

1. A. Geroldinger and I. Ruzsa, *Combinatorial Number Theory and Additive Group Theory.* Advanced Courses in Mathematics, 2009.
2. A. Granville, M. Nathanson, and J. Solymosi, *Additive Combinatorics.* CRM Proceedings and Lecture Notes, Volume 43, 2007.
3. T. Tao and V. Vu, *Additive Combinatorics.* Cambridge University Press, 2010.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.