Inverse Problems (M24)

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Inverse problems arise whenever there is a need to infer quantities of interest from indirectly measured data. Inverse problems are ubiquitous in science; they arise in physics, biology, medicine, engineering, finance and computer science (e.g., in machine learning and computer vision). Many imaging problems, such as reconstruction of medical images (computer tomography, magnetic resonance imaging, positron-emission tomography) and deblurring or denoising of microscopy and astronomy images, are also instances of inverse problems. Inverse problems typically share a feature that makes them challenging to solve in practice: they lack continuous dependence on the data and, therefore, small errors in the measurements can lead to large errors in naive reconstructions, making them useless. To deal with this issue, special *regularisation* and *Bayesian* techniques have been developed to overcome the instability by using additional a priori information about the unknown, such as smoothness or sparsity in some basis.

In this course we will present mathematical theory and algorithms for solving inverse problems using regularisation and Bayesian methods, from the classical foundations to modern state-of-the-art methods. We will apply theory and algorithms to inverse problems in imaging and engineering.

Pre-requisites

This course assumes basic knowledge in linear algebra, analysis and probability theory (e.g. Linear Analysis or Analysis of Functions and Probability and Measure). Additional knowledge in convex analysis is beneficial, but not mandatory.

Literature

1. H. W. Engl, M. Hanke and A. Neubauer. *Regularization of Inverse Problems.* Vol. 375, Springer Science & Business Media, 1996, ISBN: 9780792341574.
2. O. Scherzer, M. Grasmair, H. Grossauer, M. Haltmeier and F. Lenzen. *Variational Methods in Imaging.* Applied Mathematical Sciences, Springer New York, 2008, ISBN: 9780387309316.
3. M. Dashti and A.M. Stuart, *The Bayesian approach to inverse problems,* in: *Handbook of Uncertainty Quantification.* Springer, 2015.
4. A.M. Stuart, *Inverse problems: a Bayesian perspective.* Acta Numerica, **19**, 451-559 (2010).
5. M. Benning, M. Burger *Modern Regularization Methods for Inverse Problems*, Acta Numerica, **27**, 1-111 (2018).
6. S.L. Cotter, G.O. Roberts, A.M. Stuart, D. White, *MCMC Methods for Functions: Modifying Old Algorithms to Make Them Faster.* Statistical Science, **28**(3), 424-446 (2013).

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.