Metric Embeddings (L24)

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In the area of metric embeddings, one is mostly concerned with the following problem. Given metric spaces X and Y, is there a bi-Lipschitz embedding of X into Y, and what is the best distortion of such embeddings? In most situations of interest, X is a finite metric space and Y is a Banach space, particularly L_1, L_2 or a more general L_p space. Other types of embeddings, uniform and coarse embeddings, are also important. The aim of this course is to demonstrate the richness of this theory and the variety of the techniques (analytic, combinatorial and probabilistic) through a number of topics and major results in the field. We will also indicate the connections to other areas of mathematics (optimization, graph theory, computer science, geometry, Banach space theory). We aim to cover as many of the following topics as time permits.

Basic definitions and examples

Fréchet embeddings, Aharoni's theorem

L_1-embeddings and combinatorial optimization

Euclidean distortion, Dvoretzky's theorem, Bourgain's embedding theorem

Obstructions to embeddability: Poincaré inequalities, expander graphs

Dimension reduction in L_2, the Johnson–Lindenstrauss lemma

Impossibility of dimension reduction in L_1, Diamonds and Laakso graphs

Local theory of Banach spaces, Ribe's rigidity theorem, the Ribe programme, metric characterization of some Banach space properties

Metric theory of type and cotype, non-linear Dvoretzky theorem

Coarse embeddings into c_0 and ℓ_2; coarse embeddings of ℓ_2 into Banach spaces

Pre-requisites

Undergraduate level analysis, general topology, probability and functional analysis. In terms of the Cambridge Tripos, the Part IA Probability, the Part IB Analysis and Topology and the Part II Linear Analysis courses suffice for much of the course. In addition, knowledge of the basics of the weak and weak-star topologies in Banach spaces would be helpful.

Literature

1. Mikhail I. Ostrovskii *Metric Embeddings.* de Gruyter, 2013.

2. Jiří Matoušek *Lecture notes on metric embeddings.* online notes, 2013.

   http://kam.mff.cuni.cz/~matousek/ba-a4.pdf

Additional support

Up to four examples sheets will be provided and associated examples classes will be given. There will be a revision class in the Easter Term.