Algebraic surfaces (L24)

Dhruv Ranganathan

This course will provide an introduction to the theory of complex algebraic surfaces. The geometry of algebraic surfaces is intricate but also accessible, and provides concrete ground in which to use the basic techniques of modern algebraic geometry, including bundles, sheaves, and birational methods.

The course will begin with a rapid review of the geometry of curves and their classification, followed by a study the intersection theory of curves on surfaces, Riemann–Roch for surfaces, rational and birational maps, and the geometry of blowups and contractions. The rest of the course will proceed by analyzing several examples carefully and explicitly. We will study del Pezzo, ruled, K3, and abelian surfaces in detail, and aim to sketch the classification of surfaces of general type. Each of these examples is an entry point into active modern areas of research, spanning the minimal model program, mirror symmetry, and hyperkähler geometry.

The course will stress examples and explicit calculations and is an appropriate follow up to a first course in schemes.

Pre-requisites

The basic pre-requisite is familiarity with the basics of algebraic varieties, line bundles, and cohomology, though there will be frequent reminders to much of this material. The Part II and Part III courses in algebraic geometry, or the equivalent, will be more than sufficient background. The course may also be suitable for students who have background in complex geometry but are less familiar with scheme theory.

Literature

1. A. Beauville, *Complex Algebraic Surfaces.* 2nd edition. London Mathematical Society Student texts 34,Cambridge University Press, 1996.
2. M . Reid, *Chapters on algebraic surfaces.* Available at https://arxiv.org/abs/alg-geom/9602006, 1996.

Additional support

Detailed handwritten lecture notes will be made available online.

Four example sheets will be provided with associated examples classes. There will be a revision class during the Easter term.