Sofic Groups (L16)

Non-Examinable (Graduate Level)

Henry Bradford

A group is *sofic* if it can be “approximated” by finite groups, in a rather weak sense. In fact, the definition of soficity is so permissive that there is no group which has been proven not to satisfy it. The question of whether all groups are sofic is one of the major unsolved problems in group theory, as a positive answer to this question would also resolve famous open problems in dynamical systems; operator algebras, and ring theory. The first goal of this course is to explain what sofic groups are, and why soficity is an interesting and powerful property for a group to possess. The second goal, which will take up most of our time, is to indicate why finding a non-sofic group is so difficult. This will be achieved by showing that a wide range of natural group-theoretic conditions on a group imply soficity, which will in turn give us an opportunity to explore some of the awesome diversity of finitely generated infinite groups.

1. Definition of soficity and first properties. An application: either Gottschalk’s Surjunctivity Conjecture or Kaplansky’s Directed Finiteness Conjecture.

2. Amenable groups: definition via Følner sets and characterization in terms of invariant means; examples and closure properties; the Hausdorff-Banach-Tarski paradox and Tarski’s Theorem; sofic-by-amenable groups are sofic; elementary amenability, word growth and Grigorchuk’s first group.

3. Residual finiteness: examples including nilpotent and free groups; Abels’ group as a non-example; Mal’cev’s Theorem. Local embeddability: wreath products and symmetric enrichments; the space of marked groups; Cornulier’s construction of a non-amenable isolated sofic group.

4. Construction of a candidate for a non-sofic group.

Pre-requisites

This course should be understandable to anyone with an advanced undergraduate-level background in group theory, and some familiarity with rudiments of measure theory and functional analysis.

Literature

1. K. Juschenko, *Sofic groups,*

   https://web.ma.utexas.edu/users/juschenko/files/soficgroups.pdf

2. V.G. Pestov, *Hyperlinear and Sofic Groups: A Brief Guide.* The Bulletin of Symbolic Logic, Vol. 14, No. 4 (Dec., 2008), pp. 449–480