

Lie algebras, Vertex algebras, Shtuka (L24)

Graduate: Non-Examinable

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This is a course about semisimple Lie algebras and their generalisations.

I'll assume you are familiar with the basic structure and representation theory of the simple Lie algebras, and start again.

The point of view will be a strange mix of homotopical nonsense and down to earth calculation.

Hopefully some of the results and constructions will be new.

Pre-requisites

This course will be self contained, and proceed from the definitions. However the basics of Lie algebras are probably necessary. It would also help if you aren't scared by homological algebra—Weibel's book is a good basic reference, as is the lovely paper of Goerss-Schemmerhorn on model categories.

When (if?) we get to the chiral algebras, you will be asked to take on faith certain properties of algebraic curves, their moduli, and the stack of G -bundles on them. I'll give references in class.

Likewise for the characteristic p and spectral examples.

You are *not* supposed to have already looked at the papers and books listed below. Borcherd's paper is the definition of vertex algebras; Beilinson and Drinfeld's monograph is the definition of chiral homology. The rest are beautiful expository texts.

The last few years have seen some remarkable work on vertex algebras and quantum field theory, on $D=4$ $N=2$ SCFT, and $D=3$ $N=4$ gauged mirror symmetry. This work is related, but somewhat different to the intended topics.

Literature

Goerss, P. G., Schemmerhorn, K., Model Categories and Simplicial Methods, arXiv:math/0609537 (2006).

Weibel, C, An introduction to homological algebra, Cambridge Studies in Advanced Mathematics (38), 1994.

Borchers, R. E., Vertex algebras, Kac-Moody algebras, and the Monster, Proc Natl Acad Sci USA 83, 3068–3071 (1986).

Beilinson, A, Drinfeld V, Chiral Algebras AMS Colloquium Publications, vol. 51, 2004

Frenkel, E., Ben-Zvi, D. Vertex Algebras and Algebraic Curves. Mathematical Surveys and Monographs, vol. 88, American Mathematical Society, 2004.