

# Commutative Algebra (24 lectures, MT20)

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This course provides an introduction to the theory of commutative noetherian rings and modules over such rings. It forms a theory which is an essential ingredient in algebraic geometry, algebraic number theory and both modular and integral representation theory.

I hope to cover most of the following

- ideals for noetherian and artinian rings; Hilbert Basis Theorem and the Nullstellensatz;
- localisations and completions;
- tensor products;
- dimension theory; polynomial nature of Hilbert functions;
- projective and injective modules, resolutions and the Koszul complex;
- assorted methods from (co)homology,; Hilbert's syzygy theorem.

Four sheets of examples will be provided.

## Desirable Previous Knowledge

You will have attended a first course in ring theory, such as the IB course Groups, Rings and Modules. Experience of more advanced material such as Part II courses Galois Theory, Representation Theory, Algebraic Geometry and Number Fields is desirable but not essential.

## Books

There is no shortage of books on commutative ring theory. Notable amongst them is the 1969 classic [1], which is a clear, concise and efficient textbook aimed at beginners and with a good spread of topics. So it has remained popular. However its age and flaws are apparent. In particular many details are sketchy and important results relegated to the exercises. Sharp [6] and Kaplansky [3] are decent accounts which fill in some of the gaps. Matsumura [4] is good for the homology theory but is hard going as an introduction. Reid's book [5] is a companion to his other book on algebraic geometry, so the topics and examples can be overly specific. As in the rest of the series, [2] is encyclopedic, but it's a bit like reading a car manual. [7] is a bit dense.

1. M.F. Atiyah and I.G. Macdonald, Introduction to commutative algebra, Addison–Wesley, 1969.
2. N. Bourbaki, Commutative algebra, Elements of Mathematics, Springer, 1989.
3. I. Kaplansky, Commutative rings, Allyn and Bacon 1970.
4. H. Matsumura, Commutative ring theory, Cambridge Studies 8, CUP, 1989.
5. M. Reid, Undergraduate commutative algebra, LMS Student Texts 29, CUP 1995.
6. R.Y. Sharp, Steps in commutative algebra, LMS Student Texts 19, CUP 1990.
7. O. Zariski and P. Samuel, Commutative algebra (2 volumes), Springer GTM series.