Part III Preparatory Workshops
Michaelmas Term 2019-2020

The preparatory workshops are aimed at Part III students who have taken undergraduate courses in the relevant areas but whose courses may not have covered all the material contained in the Cambridge Undergraduate Tripos, or may have covered it from a different perspective. Of course, a few hours cannot substitute for an entire undergraduate lecture course, so the workshops aim to give some intuition and a helping hand with further study.

Certain basics will be assumed in the workshops. If you find you are missing these also, please talk to someone about your course choices as soon as possible, for example the DAMTP/DPMMS Course Director, a Part III Subject Adviser, or your college Director of Studies. The earlier you address any potential problems, the easier it will be for us to help you.

The length and format of the workshops may vary depending on the subject. In some subject areas, materials are available ahead for time and you are encouraged to consult the individual workshop descriptions contained in this booklet for details.

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Timetable 2019-2020

Subject	Date	Time	Room
Quantum Mechanics	8/10/19	2pm	MR9
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General Relativity	8/10/19	4pm	MR9
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Fluids	10/10/19	2pm	MR12
	14/10/19	2pm	MR12
Measure Theory	8/10/19	4pm	MR5
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Statistics	10/10/19	4pm	MR3
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Rings and Modules	8/10/19	2pm	MR4
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Algebraic Geometry	8/10/19	4pm	MR13
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Differential Geometry	10/10/19	4pm	MR9
	14/10/19	4pm	MR11
Number Theory	10/10/19	4pm	MR12
	14/10/19	4pm	MR12

Quantum Mechanics

The topics covered will roughly be

- Hilbert Space, Observables, etc.;
- Dirac (Bra-Ket) Notation, Schrödinger & Heisenberg Picture;
- Probabilities & Measurement;
- Harmonic Oscillator, Spin Algebra, Ladder Operators;
- Perturbation Theory Techniques.

Furthermore, you are encouraged to bring any questions, requests or specific problems about QM to the workshop, so we can talk about those in detail.

We expect you to know

You are expected to have taken basic and advanced QM courses and be familiar with the concept of states (e.g. $|\psi\rangle$), operators (e.g. $\hat{x}$), time evolution (e.g. $\exp(-it\hat{H})$), probabilities (e.g. $|\langle i|\psi\rangle|^2$), and the Schrödinger equation.

Additional resources

A set of notes can be found online at `www.damtp.cam.ac.uk/user/rb697`. It is not essential to read these, but we'll go through some of the questions in these notes as part of the workshop. A set of solutions to the questions will be uploaded on the same webpage after the workshops have taken place.

Useful for Part III courses

All Particle Physics, Quantum Fields and Strings courses, in particular Quantum Field Theory and Symmetries, Fields and Particles.

General Relativity

This workshop will serve to partially bridge the gap for those who haven't had a General Relativity course in their undergraduate degree, mostly by introducing standard notation used in GR, and giving simple examples to frame the physics behind the most mathematical aspects of the course. The topics covered will be:

- Euler-Lagrange equations and index notation;
- Special Relativity;
- equivalence principles;
- geodesic equation in General Relativity.

We expect you to know

Previous familiarity with special relativity is assumed. Although technically it will be covered, knowledge of index notation and summation convention is helpful.

Additional resources

The workshop will be mostly focused on hands-on problems and specific examples, which can be found in `http://www.damtp.cam.ac.uk/user/jfm54/part_iii_gr_workshop.pdf`. There are also notes that cover the same topics (and a bit more) on the course webpage at `http://www.damtp.cam.ac.uk/user/tong/gr.html`.

Useful for Part III courses

All Relativity and Gravitation courses but primarily General Relativity and Black Holes. Some topics (e.g. Special Relativity) are useful for the High Energy Physics courses, listed under Particle Physics, Quantum Fields and Strings.

Fluids

In this workshop we will review some basic ideas and work through some examples from the most important topics of Fluid Mechanics, that will help you revise and cover potential gaps for Part III Continuum Mechanics courses. The main methods that we will work with in the examples are vector calculus, scaling analysis and similarity solutions.

Topics from which examples will be drawn:

- Revision of suffix notation;
- Basics of Fluid Mechanics;
- Stokes flows;
- Lubrication theory;
- Vorticity;
- Boundary layers;
- Kelvin-Helmholtz instability.

N. B. Do not worry if you have not encountered suffix notation in your undergraduate degree, this workshop will include many examples and exercises. However, everything is easier the second time you see it, so we strongly advise that you look up the basic rules of suffix notation and try out a few exercises before attending the workshop.

We expect you to know

The Navier-Stokes Equations; Lagrangian and Eulerian co-ordinates; Convective derivative; Grad, Div and Curl; Stokes Theorem; The Divergence Theorem; Cylindrical and spherical polar co-ordinates.

Additional resources

Online source for learning suffix notation (page 29 onwards): `http://www.damtp.cam.ac.uk/user/sjc1/teaching/VandM/notes.pdf`
Online source for exercises (please attempt Questions 1 and 3 before the first session): `http://www.damtp.cam.ac.uk/user/examples/A1b.pdf`

Useful for Part III courses

Slow Viscous Flow, Non-Newtonian Fluid Mechanics, Fluid Dynamics of the Solid Earth, Fluid Dynamics of the Climate, Hydrodynamic Stability, Perturbation Methods.

Measure Theory

This course will cover the basics of measure theory that are required for most Part III courses in analysis. The topics covered will be:

- Construction of measures;
- Lebesgue integration and Lebesgue spaces;
- integral convergence theorems;
- product measures and Fubini's theorem;
- independence and the Borel-Cantelli lemmas;
- convergence of random variables ;
- important inequalities.

We expect you to know

Familiarity with the definition of a measure and basic calculations with measures, integration on the real line and in Euclidean space, basic notions from probability such as random variables, expectation etc.

Useful for Part III courses

Analysis of Partial Differential Equations, Ergodic Theory, Elliptic Partial Differential Equations, Advanced Probability, Stochastic Calculus and Applications, Schramm-Loewner Evolutions.

Statistics

Session 1

We will begin by reviewing some of the basic concepts in statistical modelling and inference, as well as some standard limit theorems and notions of stochastic convergence. This will lead onto a discussion of likelihood-based inference and the asymptotic properties of maximum likelihood estimators. We will develop this theory through a series of concrete and illustrative examples. If time permits, we will also cover the basics of least-squares regression and the linear model. For a more detailed treatment of these topics, consult notes for the undergraduate Principles of Statistics course. We will aim to cover:

- The strong and weak laws of large numbers and the central limit theorem;
- the likelihood and score functions, and Fisher information;
- the Cramer–Rao lower bound and efficiency;
- consistency and asymptotic normality of the MLE.

Session 2

The focal point of this session will be the normal linear model. We will study the method of ordinary least squares from both geometric and algebraic perspectives, and after reviewing some standard facts about normal, χ^2, t and F-distributions, we will explore how to carry out inference within this framework. We also aim to cover some of the basics of hypothesis testing and confidence regions. If time permits, we will also touch on some aspects of Bayesian inference and decision theory. This material forms the core of the Part IB Statistics and Part II Statistical Modelling courses. We will aim to cover:

- The least squares estimator, orthogonal projections, fitted values and residuals;
- Gauss–Markov theorem and Cochran's theorem;
- confidence sets and p-values;
- likelihood ratio tests and Wilks' theorem.

We expect you to know

- Basic probability theory, including common distributions such as binomial, Poisson, normal, multivariate normal, chi-squared etc.;
- basic linear algebra (e.g. properties of orthogonal matrices) and multivariate calculus;
- some familiarity with basic statistical procedures (terminology like Type I/II error).

Additional resources

Follow the link `http://bit.ly/2nTMB2h` to access a Dropbox folder that contains the session plan and the notes from the 2014 workshop. In addition, some background material on probability, convergence and linear regression is presented in the form of exercises. A copy of the undergraduate schedules is provided, as are some notes for the Statistical Modelling and Principles of Statistics courses.

Useful for Part III courses

Modern Statistical Methods, Topics in Statistical Theory, Statistical Learning in Practice

Rings and Modules

This workshop will cover definitions useful for Part III algebra courses, especially Algebra. We'll aim to give lots of examples, including fairly detailed looks at rings of polynomials and rings of algebraic numbers, which may help get experience for courses in algebraic geometry or algebraic number theory.

Session 1: Rings

We will try to cover:

- Ideals, maximal and prime ideals, and factorization;
- fields, principal ideal domains, unique factorization domains, integral domains;
- rings of algebraic numbers, polynomial rings;
- Noetherian rings, Hilbert's basis theorem.

There will be lots of exercises to give you practice at working with rings.

Session 2: Modules

We will aim to cover:

- Definition of a module; submodules, quotients, homomorphisms; comparison with vector spaces;
- finitely generated modules and Noetherian modules;
- the structure theorem for finitely generated modules over a principal ideal domain; applications (to abelian groups and Jordan normal form).

We expect you to know

- Basic group theory: familiarity with abstract groups, homomorphisms, quotients, direct products.
- Basic linear algebra: vector spaces and related concepts, such as linear independence, spanning sets, and bases.
- The basic definitions of rings, ring homomorphisms and ideals - although I'll provide notes on this if you need a reminder.

Useful for Part III courses

Algebra, Lie algebras and their Representations, Iwasawa Algebras, Algebraic Geometry.

Algebraic Geometry

This workshop will review some classical algebraic geometry with the aim of providing some concepts and intuitions needed in the Part III course. During the first session we will (attempt to) cover:

- Affine varieties (over an algebraically closed field);
- Zariski topology;
- the Nullstellensatz;
- rings of regular and rational functions;
- morphisms;
- projective varieties.

The second session will be used to talk about exercises from the first, answer any questions and explain any other concepts from the first week of lectures if needed.

Additional resources

This is roughly the material contained within the preliminary section of the notes from a previous version of the course available at `https://www.dpmms.cam.ac.uk/~pmhw/PtIIIAG2014.pdf`.

We expect you to know

Familiarity with point set topology and rings and ideals will be expected.

Useful for Part III courses

Algebraic Geometry.

Foundations

The workshop aims to ensure you've heard of all the necessary basics in Logic and Set Theory, ahead of the Model Theory, Infinite Games and Category Theory courses in Part III, and will attempt to set notation straight. It will loosely follow the essentials covered in the Part II course.

The first session will focus on the idea of definability, reminding attendants of syntactic and semantic aspects of first-order logic and stating core results such as the Compactness, Completeness, and Lówenheim-Skolem theorems, while the second session will be directed at Set Theory, with a recap of the ZFC axioms, ordinal and cardinal arithmetic, core notions such as that of rank and transitive set, and Hartogs' Lemma.

We expect you to know

Propositional logic (truth-tables, valuations, modus ponens, etc); familiarity with predicate logic or natural deduction and previous contact with ordinals, cardinals or axiomatic set theory are desirable.

Useful for Part III courses

Model Theory, Infinite Games, and (to a lesser extent) Category Theory.

Algebraic Topology

This workshop will introduce homology in algebraic topology with the aim of providing a gentle and intuitive introduction to the Part III courses in this area. The workshop will focus on intuition and techniques for computation. We will cover the following concepts and results:

- homotopic maps and homotopy equivalence of spaces;
- the intuition behind singular homology;
- computing with simplicial homology;
- chain complexes and exact sequences;
- the Mayer-Vietoris sequence and applications.

Few, if any, proofs will be given and some definitions will not be stated with full rigour. Where possible, I will instead illustrate concepts with examples and exercises.

We expect you to know

Standard undergraduate group theory, including the structure theorem for finitely generated abelian groups. Fluency with definitions in basic point-set topology, such as compactness and connectedness. Knowledge of the fundamental group would be good.

Useful for Part III courses

Algebraic Topology, Symplectic Topology, 3-Manifolds, Introduction to Geometric Group Theory.

Differential Geometry

In these workshops we will talk about some of the fundamental ideas in differential geometry, illustrating the abstract concepts with concrete examples. The topics we will roughly discuss are as follows:

- Smooth and topological manifolds;
- smooth functions between smooth manifolds;
- various viewpoints of the tangent space and their equivalence;
- flows on manifolds;
- relations to algebraic topology.

The sessions will be a combination of lecturing and problems for the participants to attempt, with a small problem set given at the end of each session. We will discuss the first problem set at the start of the second session.

We expect you to know

Knowledge of topological spaces is essential (e.g. Hausdorff space), as is the definition of the derivative of a function between subsets of Euclidean space as a linear map. Having taken a previous course on differential geometry is useful but definitely not essential.

Additional resources

Notes on relevant topics can be found on my website `https://minterscompactness.wordpress.com`.

Useful for Part III courses

Differential Geometry, Algebraic Topology, Symplectic Topology, Complex Manifolds, Geometric Measure Theory.

Number Theory

This workshop will cover definitions, examples and exercises useful for Number Theory courses. There will be two sessions, the first one focussing on the algebraic side and the second one on the analytic side. They can be attended independently of each other.

Session 1: Galois Theory and Number Fields

This session is useful for Algebraic Number Theory and Elliptic Curves. We aim to cover some statements in Galois theory, Galois theory for finite fields, cyclotomic fields, basic definitions for number fields and ramification of prime ideals. The plan is to first cover this theory and then give you the time to do some of the exercises and ask questions about the material.

Session 2: Complex Analysis

This session is useful for Modular Forms and L-functions and Analytic Number Theory. We aim to cover the basic definitions in complex analysis, analytic continuation, contour integrals, the Riemann zeta function, Fourier transform. The plan is to first cover this theory and then give you the time to do some of the exercises and ask questions about the material.

We expect you to know

Basic real analysis, group theory, rings and modules.

Useful for Part III courses

Algebraic Number Theory, Elliptic Curves, Modular Forms and L-functions, Analytic Number Theory.