

# Nonparametric Inference under Shape Constraints (L8)

*Non-Examinable (Graduate Level)*

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Qualitative shape restrictions such as monotonicity, convexity and log-concavity arise naturally in a variety of statistical problems, including density estimation and regression. Shape-constrained approaches to statistical inference have the potential to liberate practitioners from restrictive parametric assumptions, whilst also yielding fully automatic procedures that do not require the choice of any tuning parameters (in contrast to traditional nonparametric smoothing techniques).

This course will provide an introduction to some of the core problems in the field of nonparametric inference under shape constraints, with a particular focus on nonparametric maximum likelihood estimators and their intriguing statistical properties. If time permits, we will also survey some very recent developments in what is currently an active area of research. We will cover a selection of the following topics:

- Decreasing density estimation and the Grenander estimator: consistency and asymptotic theory.
- Log-concave density estimation: log-concave densities and their properties; log-concave projections; theory and computation of log-concave maximum likelihood estimators; extensions and applications.
- Shape-restricted regression and least squares estimation under monotonicity or convexity constraints.

## Prerequisites

Familiarity with statistics, probability and real analysis at Part IB level. Measure theory is not necessary but may be helpful. Although Topics in Statistical Theory is not a prerequisite, the two courses complement each other well.

## Literature

1. P. Groeneboom and G. Jongbloed, *Some developments in the theory of shape constrained inference*. *Statistical Science*, **33**, 473–492, 2018. Also available at <https://dutiosb.twi.tudelft.nl/~pietg/StatScienceGrJo.pdf>.
2. R. J. Samworth, *Recent progress in log-concave density estimation*. *Statistical Science*, **33**, 493–509, 2018. Also available at <http://www.statslab.cam.ac.uk/~rjs57/STS666.pdf>.
3. A. Guntuboyina and B. Sen, *Nonparametric shape-restricted regression*. *Statistical Science*, **33**, 568–594, 2018. Also available at <https://arxiv.org/pdf/1709.05707.pdf>.
4. P. Groeneboom and G. Jongbloed, *Nonparametric Inference under Shape Constraints*. Cambridge University Press, 2014.