Topological Quantum Matter (L16)

Benjamin Béri

Topologically ordered systems display quantum mechanical features that are insensitive to the local details of the system but are robustly set by its topology. Such features include "anyonic" particles beyond the boson/fermion dichotomy, a "holographic" correspondence between the system's bulk and its boundary, or a topology-dependent ground-state degeneracy. These remarkable properties, fundamentally linked to certain gauge theories, have intriguing potential applications, including the use of topological ground-state degeneracy in quantum error correction or employing so-called non-Abelian anyons in robust schemes for quantum computation.

This course is an introduction to topological order and its quantum computing applications. We begin with some basics of conventional (i.e., symmetry-breaking) order, illustrated using the quantum Ising chain. This will serve as a reference with which to compare topological order.

We then embark on our study of topological order. Concepts we shall aim to discuss include:

- ground-state degeneracy
- anyons
- the robustness of topological order
- bulk-boundary correspondence
- Majorana zero modes
- non-Abelian anyons.

We shall use the toric code and the Kitaev chain – a boson-fermion dual of the quantum Ising chain – as illustrative examples.

Of the quantum computing applications of topological order, we shall aim to discuss quantum error correction with the toric code, and topological quantum computation using non-Abelian anyons, in particular those based on Majorana zero modes.

Some of our discussions will use second quantisation, the basics of path integrals, and of quantum error correction. These shall be reviewed or introduced at the appropriate points of the course.

Prerequisites

It will be assumed that you have taken an advanced quantum mechanics course, similar to Part II Principles of Quantum Mechanics. Familiarity with the basics of band theory (e.g., from Part II Applications of Quantum Mechanics) and of quantum computation (e.g., from Part II Quantum Information and Computation) will also be useful. While not a prerequisite, having attended Part III Quantum Computation in Michaelmas term can be beneficial.

Literature

1. S. H. Simon, *Topological Quantum: Lecture Notes and Proto-Book*. Available at Steve Simon's website. Chapter 3 and Parts II, V are especially useful for this course.
2. S. Sachdev, *Quantum Phase Transitions*. CUP, 2011. Parts of chapters 1, 5, 10.
3. B. A. Bernevig with T. L. Hughes, *Topological Insulators and Topological Superconductors*. Princeton University Press, 2013. Chapter 16.

4. S. Das Sarma, M. Freedman, C. Nayak, *Majorana zero modes and topological quantum computation.* npj Quantum Inf. **1**, 15001 (2015).

5. A. Altland and B. Simons, *Condensed Matter Field Theory.* CUP, 2010. Chapters 2, 3 for background on second quantisation and path integrals.

Additional support

Three example sheets will be provided and three associated examples classes will be given. There will be a one-hour revision class in the Easter Term.