Symmetries, Particles and Fields (M24)

Ben Allanach

Lie groups and Lie algebras are important in the construction of quantum field theories which describe interactions between known particles. Particle states and quantum fields are described in terms of irreducible representations of the Poincaré group, a Lie group. Gauge theories, which describe many of the interactions in the Standard Model of particle physics, rely on Lie groups.

After some basic properties and preliminaries, we introduce Lie matrix groups which rely on continuous parameters. Differentially, these act as a Lie algebra. The exponential map connects the Lie algebra to the Lie group.

We then introduce representations in terms of square matrices, describing how to construct various new representations in terms of old ones.

The group of rotations in three-dimensional space $SO(3)$ is examined, along with $SU(2)$ and the connection to angular momentum states in quantum theory. Representations of each are covered.

The relativistic symmetries (Lorentz group and Poincaré group in four dimensions) are studied from the point of view of their group elements and Lie algebras. We shall cover irreducible induced representations of the Poincaré group, which are used to describe one-particle states and fields.

Analysis of compact simple Lie algebras and their finite representations comes from mapping them to a geometrical picture involving roots and weights via the Cartan matrix. An overview of the results of the Cartan classification of simple Lie algebras is included.

An application in terms of representations of a global $SU(3)_F$ flavour symmetry explains some features of the spectrum of hadronic particles. Further properties of the spectrum lead one to introduce an additional local $SU(3)_c$ colour symmetry leading a particular gauge theory called quantum chromodynamics.

We shall close by covering gauge theory: first $U(1)$, then for general groups and general representations of matter.

Prerequisites

Linear algebra including direct sums and tensor products of vector spaces. Special relativity and quantum theory, including orbital angular momentum theory and Pauli spin matrices.

Literature

1. *Symmetries, Particles and Fields*, B.C. Allanach, Kindle Direct Publishing, 2021 will form the printed and extended course notes.

2. *Symmetries and Group Theory in Particle Physics*, G. Costa and G. Fogli, Springer, 2012 has nice geometrical aspects.

3. For a more basic introduction to the use of groups in physics, *Group Theory in a Nutshell for Physicists*, Zee, Princeton University Press, 2016.

4. *Representations and Physics*, 2nd edition, Taylor and Francis, 1998.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.