

# Algebraic Number Theory (L24)

Dr H. Wiersema

This course constitutes a second course in algebraic number theory and will cover various topics in the subject. One of the main topics we will discuss is global class field theory, which aims to classify abelian extensions of number fields. We will not prove the main statements, but we will discuss both the ideal and the idele theoretic version of class field theory. After introducing the necessary concepts in each version, we will see how these two are compatible.

We will study other objects central to current research in number theory, such as  $\zeta$ -functions and  $L$ -functions. Moreover, we will discuss density and density results like the Chebotarev density theorem.

There is plenty of literature in this area. We focus on the books [3], [4] and [7].

Topics likely to be included (not in order):

- Dedekind  $\zeta$ -functions, analytic class number formula.
- $L$ -functions.
- Ideles and adèles.
- Statements of global class field theory.
- Dirichlet density, Chebotarev density theorem.

## Prerequisites

Galois Theory, Number Fields, Local Fields.

## Literature

1. J.W.S. Cassels and A. Fröhlich (eds.), *Algebraic Number Theory*, Academic Press, 1967.
2. N. Childress, *Class Field Theory*, Springer New York, 2008.
3. D.A. Cox, *Primes of the Form  $p = x^2 + ny^2$  : Fermat, Class Field Theory, and Complex Multiplication*, John Wiley & Sons, 1989.
4. G.J. Janusz, *Algebraic Number Fields*, Academic Press, 1973.
5. S. Lang, *Algebraic Number Theory*, Springer GTM 110, 1970.
6. D. Marcus, *Number Fields*, Springer-Verlag, 1977.
7. J. Neukirch, *Algebraic Number Theory*, Springer-Verlag, 1999.
8. J. Neukirch, *Class Field Theory. The Bonn Lectures*. Online edition. Available via <https://www.mathi.uni-heidelberg.de/~schmidt/Neukirch-en/>
9. H.P.F. Swinnerton-Dyer, *A Brief Guide to Algebraic Number Theory* (London Mathematical Society Student Texts), Cambridge University Press, 2001.

## **Additional support**

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term.